1. $f(x) = x^2 + 1 \qquad g(x) = 3x - 4$

$$g(f(x)) = g(x^2+1)$$
$$= 3(x^2+1) - 4$$
$$= 3x^2 + 3 - 4$$
$$= 3x^2 - 1$$

(A)

2. $P(5, 12) \qquad y = x^2 - 4x + 7$

$$\frac{dy}{dx} = 2x - 4$$

when $x = 5$,

$$\frac{dy}{dx} = 2(5) - 4$$
$$m = 6$$

(B)

3. $2x^2 + 4x + 5 = 0$

$a = 2 \quad b = 4 \quad c = 5$

$$b^2 - 4ac = (4)^2 - 4(2)(5)$$
$$= 16 - 40$$
$$= -24$$

(B)

4. $y = 4\cos 2x - 1$

$max = 4 - 1 = 3$
$min = -4 - 1 = 5$

moved down 1

2 waves in $360°$ (2π)
1 wave in $180°$ (π)

$\Rightarrow$ (A)

5. $5x + 3y - 6 = 0 \qquad (-2, -1)$

$$3y = -5x + 6$$
$$y = \frac{-5}{3}x + 2$$

$$y - b = m(x - a)$$
$$y + 1 = -\frac{5}{3}(x + 2)$$
$$3y + 3 = -5x - 10$$
$$5x + 3y + 13 = 0$$

(D)

6. $x = 2$

$x = 2$	1	3	−5	−6
		2	10	10
	1	5	5	4

The remainder is 4 (C)

7. $\int x(3x+2)\,dx$

$$= \int (3x^2 + 2x)\,dx$$
$$= x^3 + x^2 + C$$

(B)

8. $u_{n+1} = 0{\cdot}1u_n + 8 \qquad u_1 = 11$

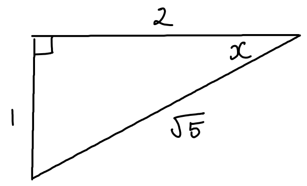
$$11 = 0{\cdot}1u_0 + 8$$
$$3 = 0{\cdot}1u_0$$
$$u_0 = 30$$

a limit exists as $-1 < 0{\cdot}1 < 1$

$\Rightarrow$ Statement (2) true

(C)

9.



$\sin x = \frac{1}{\sqrt{5}}$ $\cos x = \frac{2}{\sqrt{5}}$

$\sin 2x = 2\sin x \cos x$

$= 2 \cdot \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}}$

$= \frac{4}{5}$ (A)

10. $\cos(270 - a)^\circ = \cos 270^\circ \cos a^\circ + \sin 270^\circ \sin a^\circ$

$= 0 . \cos a^\circ + (-1) \sin a^\circ$

$= 0 - \sin a^\circ$

$= - \sin a^\circ$ (D)

11. $y = - f(x - k)$

reflected on x-axis ↑ shifted right ↑

(B)

12. $f = 3\underline{i} + 2\underline{k}$ $g = 2\underline{i} + 4\underline{j} + 3\underline{k}$

$f + g = 5\underline{i} + 4\underline{j} + 5\underline{k}$

$|f + g| = \sqrt{5^2 + 4^2 + 5^2}$

$= \sqrt{66}$ (C)

13. $x^2 - 7x + 12$ cannot equal zero

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x = 3, 4$

$\Rightarrow x \neq 3, 4$ (A)

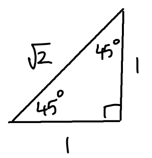
14. $|\underline{a}| = 3$ $|\underline{b}| = 2$ $\underline{a} . \underline{b} = 5$

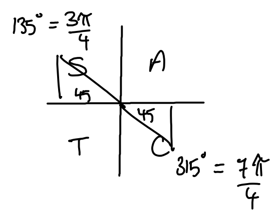
$\underline{a} . (\underline{a} + \underline{b}) = \underline{a} . \underline{a} + \underline{a} . \underline{b}$

$= 3^2 + 5$

$= 14$ (B)

15. $\tan\left(\frac{x}{2}\right) = -1$





$\frac{x}{2} = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = \frac{3\pi}{2}, \frac{7\pi}{2}$ $\left(\frac{7\pi}{2} > 2\pi\right)$

$\Rightarrow x = \frac{3\pi}{2}$ (C)

16. $\int (1 - 6x)^{-1/2} \, dx$

$= \frac{(1-6x)^{1/2}}{\frac{1}{2} . (-6)} + c$

$= \frac{(1-6x)^{1/2}}{-3} + c$

$= -\frac{1}{3}(1-6x)^{1/2} + c$ (C)

17. $(-2,0)$ $(0,0)$ $(1,3)$

roots

$y = kx(x+a)^2$

$y = kx(x+2)^2$

root $x=0$ root $x=-2$

when $x=1$, $y=3$

$3 = k(1)((1)+2)^2$

$3 = k(3)^2$

$3 = 9k$

$k = \frac{1}{3}$

$\Rightarrow$ $a = 2$, $k = \frac{1}{3}$ (C)

18. $y = \sin(x^2-3)$

$\frac{dy}{dx} = \cos(x^2-3) \cdot 2x$

$= 2x\cos(x^2-3)$ (D)

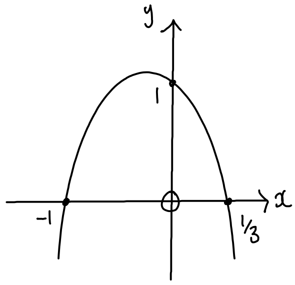
19. $1 - 2x - 3x^2 > 0$

$(1-3x)(1+x) > 0$

for roots either $1-3x = 0$ or $1+x=0$

$3x = 1$ $\qquad x = -1$

$x = \frac{1}{3}$



for $1-2x-3x^2 > 0$

$-1 < x < \frac{1}{3}$ (B)

20. From graph $m = 2$ & $c = 0$

$\Rightarrow$ $\log_3 y = 2x$

$y = 3^{2x}$

$y = (3^2)^x$

$y = 9^x$ (D)

21.

$$2x^2 + 12x + 1$$
$$= 2(x^2 + 6x) + 1$$
$$= 2\left[(x+3)^2 - 9\right] + 1$$
$$= 2(x+3)^2 - 18 + 1$$
$$= 2(x+3)^2 - 17$$

22. (a)

$$x^2 + y^2 + 2x + 4y - 27 = 0$$

$C_1(-1,-2)$ $\quad r_1 = \sqrt{(-1)^2 + (-2)^2 - (-27)}$

$$r_1 = \sqrt{1 + 4 + 27}$$
$$r_1 = \sqrt{32}$$

(b)

$C_1(-1,-2)$ $\quad P(3,2)$

$$m_r = \frac{2-(-2)}{3-(-1)}$$
$$m_r = \frac{4}{4}$$
$$m_r = 1$$

$\Rightarrow m_T = -1$ as $m_1 m_2 = -1$ for $\perp$ lines

$P(3,2)$ $\quad m = -1$

$$y - b = m(x-a)$$
$$y - 2 = -1(x-3)$$
$$y - 2 = -x + 3$$
$$y = 5 - x$$

(c)

$r_1 = \sqrt{32}$
$\quad = 4\sqrt{2}$

$r_2 = 2\sqrt{2}$
$(= \sqrt{8})$

$C_2(10,-1)$

$$(x-10)^2 + (y+1)^2 = 8$$
$$x^2 - 20x + 100 + y^2 + 2y + 1 - 8 = 0$$
$$x^2 + y^2 - 20x + 2y + 93 = 0$$

PROVED

(d)

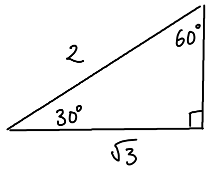
for points of intersection $y = y$

$$x^2 + (5-x)^2 - 20x + 2(5-x) + 93 = 0$$
$$x^2 + 25 - 10x + x^2 - 20x + 10 - 2x + 93 = 0$$
$$2x^2 - 32x + 128 = 0$$
$$x^2 - 16x + 64 = 0$$
$$(x-8)(x-8) = 0$$

$x = 8$ $\quad$ one solution $\Rightarrow$ tangency

23. (a) $\sqrt{3}\sin x^\circ - \cos x^\circ$

$k\sin(x-a)^\circ = k\sin x^\circ\cos a^\circ - k\cos x^\circ \sin a^\circ$



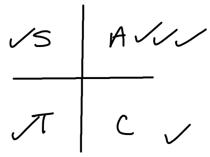
$k\cos a^\circ = \sqrt{3}$
$k\sin a^\circ = 1$

$\tan a^\circ = \dfrac{k\sin a^\circ}{k\cos a^\circ}$

$\tan a^\circ = \dfrac{1}{\sqrt{3}}$

$k^2 = (\sqrt{3})^2 + (1)^2$
$k^2 = 3+1$
$k^2 = 4$
$k = 2$

acute angle $= 30^\circ$



a° is in 1st quadrant
$\Rightarrow a^\circ = 30^\circ$

$\sqrt{3}\sin x^\circ - \cos x^\circ = 2\sin(x-30)^\circ$

(b) $4 + 5\cos x^\circ - 5\sqrt{3}\sin x^\circ$

$= 4 - 5(\sqrt{3}\sin x^\circ - \cos x^\circ)$

$= 4 - 5(2\sin(x-30)^\circ)$

$= 4 - 10\sin(x-30)^\circ$

max value $= 10 + 4$
$= 14$

24. (a) (i) $A(-7,-8,1)$ $T(3,2,5)$ $B(18,17,11)$

$\vec{AT} = \begin{pmatrix}10\\10\\4\end{pmatrix} = 2\begin{pmatrix}5\\5\\2\end{pmatrix}$ $\quad \vec{TB} = \begin{pmatrix}15\\15\\6\end{pmatrix} = 3\begin{pmatrix}5\\5\\2\end{pmatrix}$

$\vec{AT} = \frac{2}{3}\vec{TB}$

$\Rightarrow \vec{AT} \parallel \vec{TB}$ and share common point T

so A, T and B are collinear.

(ii) T divides AB in ratio 2 : 3

(b) $C(x,0,0)$

If $\vec{TB} \perp \vec{TC}$ then $\vec{TB}.\vec{TC} = 0$

$\vec{TC} = \begin{pmatrix}x-3\\0-2\\0-5\end{pmatrix} = \begin{pmatrix}x-3\\-2\\-5\end{pmatrix}$

$\vec{TB}.\vec{TC} = 15.(x-3) + (-2).15 + (-5).6$

$0 = 15x - 45 - 30 - 30$

$15x = 105$

$x = 7$

$\Rightarrow C(7,0,0)$