ALGEBRAIC OPERATIONS

By the end of this set of exercises, you should be able to

(a) multiply algebraic expressions involving brackets

(b) factorise algebraic expressions

(c) factorise trinomial expressions

ALGEBRAIC OPERATIONS

A. Multiplying Algebraic Expressions Involving Brackets

Exercise 1

1. Write these without brackets:

(a) $6(x + 2)$	(b) $3(a + 1)$	(c) $5(y - 4)$	(d) $7(t - 1)$
(e) $10(x - 10)$	(f) $2(2 + x)$	(g) $3(4 + y)$	(h) $6(5 - w)$
(i) $8(1 - c)$	(j) $15(2 - h)$	(k) $3(x + y)$	(l) $9(a - c)$
(m) $4(2 - x)$	(n) $11(e - f)$	(o) $1(1 - y)$	(p) $1(y - 1)$

2. Remove the brackets:

(a) $3(2x + 4)$	(b) $2(4a + 3)$	(c) $5(1 + 2y)$	(d) $6(3 - 3x)$
(e) $7(2w - 4)$	(f) $c(x + 5)$	(g) $d(v + 3)$	(h) $g(h - 1)$
(i) $s(r - 4)$	(j) $m(n + 10)$	(k) $x(v + w)$	(l) $a(x + r)$
(m) $x(a - y)$	(n) $a(a + b)$	(o) $r(r - s)$	(p) $r(r - 1)$
(q) $a(1 - a)$	(r) $x(x - 8)$	(s) $x(x + 3y)$	(t) $w(3w - 1)$
(u) $x(5x - 3)$	(v) $a(7x - 5a)$	(w) $m(4m + 8n)$	(x) $v(27 - 2v)$

3. Multiply out the brackets:

(a) $2(x + y + 4)$	(b) $7(x + y + 1)$	(c) $5(x - y - 6)$
(d) $6(x + 2y + 5)$	(e) $10(4x - y + z)$	(f) $9(6a - 2b + 1)$
(g) $x(3x + 5y + z)$	(h) $2a(3a - 4b + c)$	(i) $s(s^2 + 3)$
(j) $x(x^2 + 1)$	(k) $y(y^2 - 1)$	(l) $c(c^2 - 6)$
(m) $w(w^2 + w)$	(n) $a(a^2 - a)$	(o) $x(x^3 - 2x^2)$

Exercise 2A

1. Multiply out these brackets:

(a) $(x + 1)(x + 5)$	(b) $(x + 2)(x + 3)$	(c) $(x + 5)(x + 6)$
(d) $(x + 3)(x + 7)$	(e) $(x + 4)(x + 4)$	(f) $(x + 1)(x + 1)$
(g) $(a + 1)(a + 8)$	(h) $(s + 11)(s + 10)$	(i) $(w + 4)(w + 100)$

2. Multiply:

(a) $(x - 3)(x - 1)$	(b) $(x - 4)(x - 2)$	(c) $(x - 7)(x - 8)$
(d) $(a - 2)(a - 5)$	(e) $(b - 7)(b - 7)$	(f) $(c - 3)(c - 2)$
(g) $(v - 10)(v - 10)$	(h) $(w - 6)(w - 3)$	(i) $(z - 1)(z - 1)$

3. Multiply:

(a) $(x + 5)(x + 1)$	(b) $(c - 4)(c - 2)$	(c) $(s - 6)(s + 3)$
(d) $(a - 7)(a - 5)$	(e) $(v + 9)(v + 9)$	(f) $(q - 6)(q + 2)$
(g) $(r + 6)(r - 2)$	(h) $(w - 8)(w + 8)$	(i) $(x + 1)(x - 1)$
(j) $(d - 3)(d - 3)$	(k) $(a - 6)(a + 11)$	(l) $(z - 10)(z + 11)$

4. Multiply:

(a) $(2x + 3)(2x - 3)$	(b) $(5c - 1)(5c + 1)$	(c) $(2s - 1)(2s + 3)$
(d) $(2a - 3)(2a - 1)$	(e) $(v + 1)(4v - 3)$	(f) $(3q - 4)(2q + 3)$
(g) $(4r - 2)(5r + 3)$	(h) $(4w - 5)(2w + 5)$	(i) $(10x + 1)(10x - 1)$
(j) $(2 - d)(1 - d)$	(k) $(4 - p)(3 + 2p)$	(l) $(1 - 3p)(1 - 2p)$

5. Multiply out:

(a) $(x + 2)^2$	(b) $(y + 4)^2$	(c) $(z + 3)^2$	(d) $(t + 10)^2$
(e) $(x - 1)^2$	(f) $(y - 6)^2$	(g) $(z - 2)^2$	(h) $(t - 8)^2$
(i) $(a + b)^2$	(j) $(g + h)^2$	(k) $(r - s)^2$	(l) $(e - f)^2$
(m) $(3x + 1)^2$	(n) $(4x - 3)^2$	(o) $(x + 3y)^2$	(p) $(a - 4b)^2$
(q) $(4a + b)^2$	(r) $(5c + d)^2$	(s) $(5p + 2q)^2$	(t) $(2x - 3y)^2$

Exercise 2B

Multiply out the brackets and simplify:

1. $(x + 1)(x^2 + 3x + 1)$	2. $(x + 2)(x^2 - 4x + 1)$
3. $(w - 3)(w^2 + w - 2)$	4. $(z - 1)(z^2 - 5z - 1)$
5. $(v + 2)(2v^2 + v + 5)$	6. $(a - 5)(5a^2 - 10a - 20)$
7. $(m + 2)^3$	8. $(n - 1)^3$
9. $(x + {}^1/_x)^2$	10. $(x - {}^1/_x)^2$

B. Factorising Algebraic Expressions – The Common Factor

Exercise 3

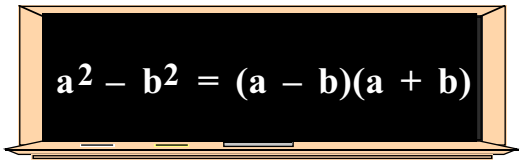
1. Factorise the following by taking out the common factors:

(a) $4a + 4b$	(b) $7v + 7w$	(c) $3x - 3y$	(d) $6c - 6d$
(e) $2r + 4s$	(f) $9m - 12n$	(g) $av + aw$	(h) $pq - pr$
(i) $bx + b$	(j) $ax^2 + a$	(k) $x^2 + dx$	(l) $y^2 - yz$
(m) $a^2 + a$	(n) $t^2 - t$	(o) $h^3 + h^2$	(p) $m^3 - m^2$
(q) $ab + bt$	(r) $mn - nr$	(s) $8x + 12y$	(t) $35p - 21q$
(u) $2a^2 + 8ab$	(v) $12ab - 9ac$	(w) $pqr + pqs$	(x) $8c^2 - 2c$

2. Factorise:

(a) $am - bm$	(b) $20 - 5w$	(c) $d - d^2$	(d) $yz + z$
(e) $pr - pu$	(f) $2mn + mp$	(g) $6cd - 4ce$	(h) $9pq - 12pr$
(i) $8a^2 + 6a$	(j) $15x^2 - 6xy$	(k) ${}^1/_2x + {}^1/_2y$	(l) $pq + {}^1/_2sq^2$
(m) $10a^2b + 8ab^2$	(n) ${}^1/_2 + {}^1/_2x$	(o) ${}^1/_2v - {}^3/_2$	(p) $2\pi rh + 2\pi r^2$
(q) $6a + 3b - 12c$	(r) $mn - mp + m^2$	(s) $3x^2 - 2xy + 6x$	(t) $25x^2 - 5x^2y$

C. Difference of Two Squares



Exercise 4

1. Factorise:

(a) $x^2 - y^2$	(b) $p^2 - q^2$	(c) $d^2 - e^2$	(d) $x^2 - 3^2$
(e) $y^2 - 4^2$	(f) $t^2 - 5^2$	(g) $5^2 - t^2$	(h) $9^2 - q^2$
(i) $1 - v^2$	(j) $x^2 - 4$	(k) $k^2 - 25$	(l) $n^2 - 36$
(m) $d^2 - 100$	(n) $e^2 - 121$	(o) $144 - y^2$	(p) $49 - x^2$
(q) $x^2 - 1$	(r) $1 - y^2$	(s) $81 - a^2$	(t) $10000 - b^2$

2. Factorise:

(a) $9a^2 - 4$	(b) $4b^2 - 25$	(c) $16c^2 - 1$	(d) $25d^2 - 36$
(e) $9e^2 - 16$	(f) $25f^2 - 81$	(g) $4g^2 - h^2$	(h) $j^2 - 25k^2$
(i) $64m^2 - 49n^2$	(j) $4p^2 - 9q^2$	(k) $81r^2 - 1$	(l) $1 - 64s^2$
(m) $121 - 16t^2$	(n) $100u^2 - 121v^2$	(o) $10000w^2 - 1$	(p) $25x^2 - 49y^2$

3. Factorise these, by taking out the common factor first:

(a) $2a^2 - 18$	(b) $5b^2 - 5$	(c) $6c^2 - 54$	(d) $4d^2 - 16$
(e) $7e^2 - 7g^2$	(f) $6p^2 - 24q^2$	(g) $10x^2 - 90y^2$	(h) $12u^2 - 12v^2$
(i) $am^2 - an^2$	(j) $ka^2 - 25kb^2$	(k) $nr^2 - 81nq^2$	(l) $d^3 - 49d$
(m) $64b - b^3$	(n) $2u^3 - 32u$	(o) $12w^3 - 27w$	(p) $11x^5 - 11x^3$

D. Trinomial Expressions

Exercise 5

Factorise the expressions:

1. $x^2 + 3x + 2$	2. $x^2 + 5x + 6$	3. $x^2 + 2x + 1$
4. $y^2 + 6y + 5$	5. $y^2 + 11y + 10$	6. $y^2 + 8y + 7$
7. $v^2 + 9v + 20$	8. $v^2 + 7v + 10$	9. $v^2 + 6v + 8$
10. $w^2 - 2w + 1$	11. $w^2 - 4w + 4$	12. $w^2 - 6w + 9$
13. $a^2 - 3a + 2$	14. $a^2 - 7a + 12$	15. $a^2 - 8a + 7$
16. $c^2 - 13c + 42$	17. $c^2 - 11c + 24$	18. $c^2 - 10c + 9$
19. $s^2 + 12s + 36$	20. $s^2 - 12s + 36$	21. $s^2 + 14s + 49$
22. $z^2 - 14z + 49$	23. $z^2 + 13z + 36$	24. $z^2 - 13z + 36$
25. $b^2 + 37b + 36$	26. $b^2 - 37b + 36$	27. $b^2 - 18b + 81$

cont'd

28. $p^2 + 6p + 9$
29. $p^2 - 7p - 8$
30. $p^2 + 4p + 4$
31. $m^2 + 11m + 30$
32. $m^2 + m - 12$
33. $m^2 - m - 6$
34. $n^2 - 8n + 15$
35. $n^2 + 3n - 10$
36. $n^2 - 3n - 4$
37. $r^2 - 2r - 8$
38. $r^2 + 5r - 6$
39. $r^2 + 12r + 36$
40. $e^2 - 5e - 14$
41. $e^2 + 7e + 12$
42. $e^2 - e - 56$
43. $g^2 - 7g + 12$
44. $g^2 - g - 6$
45. $g^2 - g - 12$
46. $k^2 - 4k - 5$
47. $k^2 + k - 6$
48. $k^2 + 2k - 35$
49. $y^2 + 4y - 12$
50. $y^2 + 3y - 18$
51. $y^2 - 3y - 28$
52. $x^2 - 3x - 40$
53. $x^2 - 2x - 15$
54. $x^2 + 11x + 30$
55. $v^2 - 9v + 8$
56. $v^2 + 5v - 24$
57. $v^2 - 5v - 24$
58. $w^2 + 2w - 24$
59. $w^2 - 2w - 24$
60. $w^2 + 10w - 24$
61. $a^2 - 10a - 24$
62. $a^2 + 23a - 24$
63. $a^2 - 23a - 24$
64. $b^2 + 7b - 30$
65. $b^2 - 4b - 45$
66. $b^2 - 7b - 18$
67. $c^2 + 15c + 56$
68. $c^2 - 15c + 54$
69. $c^2 + 18c + 81$
70. $d^2 - 12d - 28$
71. $d^2 + 49d - 50$
72. $d^2 - 51d + 50$
73. $a^2 + 2ab + b^2$
74. $x^2 - 2xy + y^2$
75. $p^2 - pq - 2q^2$

Exercise 6

Factorise these expressions:

1. $2x^2 + 7x + 3$
2. $2y^2 + 5y + 3$
3. $3w^2 + 7w + 2$
4. $10a^2 + 17a + 3$
5. $6b^2 + 7b + 2$
6. $6c^2 + 7c + 1$
7. $3d^2 + 14d + 15$
8. $10m^2 + 19m + 6$
9. $2p^2 - 7p + 3$
10. $12n^2 - 8n + 1$
11. $2q^2 - 5q + 3$
12. $6x^2 - 13x + 6$
13. $8s^2 - 14s + 5$
14. $9r^2 - 24r + 16$
15. $12g^2 - 23g + 10$
16. $3k^2 - 5k + 2$
17. $3y^2 - 2y - 8$
18. $3w^2 - 5w - 2$
19. $6u^2 - 5u - 6$
20. $5v^2 + 4v - 1$
21. $2x^2 + x - 1$
22. $3d^2 - 2d - 1$
23. $8a^2 + 2a - 3$
24. $12y^2 - 11y - 5$
25. $4p^2 - 11p + 6$
26. $15 - 7x - 2x^2$
27. $5 + 11x - 12x^2$
28. $1 - 8x + 16x^2$
29. $1 - 3x - 18x^2$
30. $4p^2 - 7pq - 2q^2$

Exercise 7 (Miscellaneous Examples on Factorisation)

Factorise FULLY:

1. $4x + 12y$
2. $a^2 - 81$
3. $w^2 + 10w + 25$
4. $y^2 - y$
5. $v^2 - v - 12$
6. $1 - b^2$
7. $u^2 + 12u + 36$
8. $ap - aq + ar$
9. $7x^2 - 28$
10. $w^2 - r^2$
11. $h^2 - 11h$
12. $x^2 - 2x + 1$
13. $t^2 - 1$
14. $t^2 - t$
15. $a^2 - 2a - 3$
16. $3c^2 - 48$
17. $5d^2 - 20d$
18. $a^4 - a^3$
19. $2s^2 + 3s - 5$
20. $x^2 - 12x + 36$
21. $16y^2 + 8y + 1$
22. $49 - g^2$
23. $36 - 4r^2$
24. $14z - 7z^2$
25. $25 - 9g^2$
26. $2b^2 - b - 1$
27. $6x^2 + 7x - 3$
28. $11u^2 - 44v^2$
29. $21u^2 + 28v^2$
30. $25p^2 - 10p + 1$
31. $3m^2n - 6mn^2$
32. $1 - 2n + n^2$
33. $27 - 6s - s^2$
34. $3a^3 - 48a$
35. $8n^2 + 8n - 6$
36. $8n^2 - 8n + 2$
37. $5r^2 + 5r - 10$
38. $4w^2 + 14w - 8$
39. $7x - 63x^3$
40. $9x + 27x^3$
41. $xy^2 - xz^2$
42. $2e^2 - 11e - 21$
43. $x^4 - 1$
44. $2 - 4q + 2q^2$
45. $g^2 + gh - 6h^2$
46. $2k^2 + 3\pi Rk + \pi^2 R^2$
47. $a^2 - a^6$
48. $k^4 + 2k^2 + 1$
49. $2a^4 - 2a^2 - 12$
50. $b^5 - 81b$
51. $3x^4 + 5x^2 - 2$
52. $9x^4 - 24x^2 + 16$
53. $2x^4 - x^2 - 3$
54. $1 - y^8$

Checkup for Algebraic Operations

1. Remove the brackets:

(a) $3(4x + 1)$	(b) $y(a - y)$	(c) $v(v - 1)$	(d) $7w(2w - 5)$
(e) $6(3x + 2y - 1)$	(f) $c(c^2 + c - 1)$	(g) $3d(4a + 3b)$	(h) $g(h^2 - g^2)$
(i) $6x(3x + 2y - 1)$	(j) $c^2(c^2 + c - 4)$	(k) $ab(3a + 4b)$	(l) $2pq(5 - q)$

2. Multiply out the brackets:

(a) $(x + 1)(x + 7)$	(b) $(x - 2)(x - 3)$	(c) $(x + 5)(x - 6)$
(d) $(x - 3)(x + 9)$	(e) $(x + 1)^2$	(f) $(x - 2)^2$
(g) $(5x - 1)(4x + 7)$	(h) $(2x - 1)(6x - 3)$	(i) $(2x - 4)(3x + 1)$
(j) $(2x - 3)^2$	(k) $(x - 2)(4x^2 - 3x + 2)$	(l) $(x - 3)^3$

3. Factorise fully:

(a) $9m - 9n$	(b) $6a - 15b$	(c) $y - y^2$
(d) $14p^2 + 6q$	(e) $3pr + pu$	(f) $4p^2 + 6pq - 2p$
(g) $6x + 30y - 15z$	(h) $9pq - 12pr$	(i) $r^2 - s^2$
(j) $81 - q^2$	(k) $16r^2 - 49$	(l) $2b^2 - 32$
(m) $20w^3 - 45w$	(n) $y^2 - 3y + 2$	(o) $a^2 - 7a - 30$
(p) $y^2 + y - 6$	(q) $24 + 10r - r^2$	(r) $x^2 - 14x + 49$
(s) $6p^2 - 17p + 12$	(t) $4x^2 + 4x + 1$	(u) $2q^2 - 2q - 144$
(v) $2x^2 + 3xy - 2y^2$	(w) $6a^4 + 2a^2 - 4$	(x) $5y^4 - 12y^2 - 9$