HIGHER 2007 PAPER 2 SOLUTIONS

1. a) $Q(0,2,2)$

b) $\underline{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\quad \underline{q} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

c) $\underline{p}\cdot\underline{q} = 0.1 + 1.2 + 1.1$

$= 3$

$|\underline{p}| = \sqrt{0^2 + 1^2 + 1^2}$

$= \sqrt{2}$

$|\underline{q}| = \sqrt{1^2 + 2^2 + 1^2}$

$= \sqrt{6}$

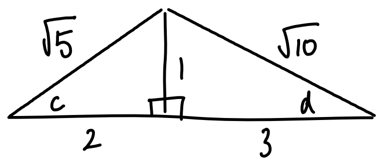
$\underline{p}\cdot\underline{q} = |\underline{p}||\underline{q}| \cos P\hat{O}Q$

$\cos P\hat{O}Q = \dfrac{3}{\sqrt{2}.\sqrt{6}}$

$P\hat{O}Q = \cos^{-1}\left(\dfrac{3}{\sqrt{2}.\sqrt{6}}\right)$

$P\hat{O}Q = 30°$

2. a)



$\sin(c+d) = \sin c \cos d + \cos c \sin d$

$= \dfrac{1}{\sqrt{5}}\cdot\dfrac{3}{\sqrt{10}} + \dfrac{2}{\sqrt{5}}\cdot\dfrac{1}{\sqrt{10}}$

$= \dfrac{3}{\sqrt{50}} + \dfrac{2}{\sqrt{50}}$

$= \dfrac{5}{\sqrt{50}}$

$= \dfrac{5}{5\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}}$

b) (i) $\sin 2c = 2\sin c \cos c$

$= 2.\dfrac{1}{\sqrt{5}}.\dfrac{2}{\sqrt{5}}$

$= \dfrac{4}{5}$

(ii) $\cos 2d = 2\cos^2 d - 1$ *

$= 2\left(\dfrac{3}{\sqrt{10}}\right)^2 - 1$

$= 2.\dfrac{9}{10} - 1$

$= \dfrac{18}{10} - 1$

$= \dfrac{8}{10}$

$= \dfrac{4}{5}$

* any of 3 formulae will work

3. For points of intersection $y = y$,

$$x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$$

$$x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$$

$$5x^2 - 10x + 5 = 0$$

$$x^2 - 2x + 1 = 0$$ * or use discriminant

$$(x-1)(x-1) = 0$$

$$x = 1 \text{ twice}$$

$\Rightarrow$ line is tangent to circle

when $x = 1$,

$$y = 6 - 2(1)$$
$$= 4$$

Point of contact $(1, 4)$

4. a) $y = 2\sin 3x - 1$

$a = 2$, $b = 3$, $c = -1$

b) $2\sin 3x - 1 = 0$

$$\sin 3x = \frac{1}{2}$$

$$3x = 30^\circ, 150^\circ$$

$$x = 10^\circ, 50^\circ, \ldots$$

$\Rightarrow P(50^\circ, 0)$ by inspection

5. a)

$$y = \frac{1}{2}x^2 - 8x + 34$$

$$\frac{dy}{dx} = x - 8$$

at Q, $m = 4$

$$\Rightarrow \quad x - 8 = 4$$

$$x = 12$$

$$y = \frac{1}{2}(12)^2 - 8(12) + 34$$

$$= 72 - 96 + 34$$

$$= 10$$

$$\Rightarrow \quad Q(12, 10)$$

b) P will have same y-coordinate as Q.

$$\Rightarrow \quad \frac{1}{2}x^2 - 8x + 34 = 10$$

$$\frac{1}{2}x^2 - 8x + 24 = 0$$

$$x^2 - 16x + 48 = 0$$

$$(x-4)(x-12) = 0$$

$$\Rightarrow \quad x = 4, 12$$

Hence $P(4, 10)$

c)* $C(8, y)$ by symmetry

gradient of curve at $Q = 4$

$$\Rightarrow \quad m_{CQ} = -\frac{1}{4}$$

$$-\frac{1}{4} = \frac{10 - y}{12 - 8}$$

$$-\frac{1}{4} = \frac{10 - y}{4}$$

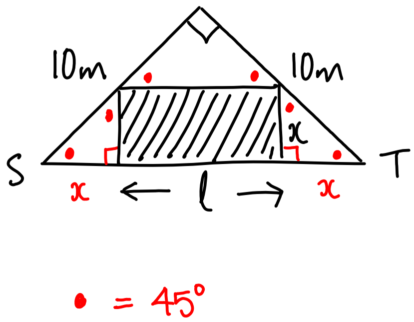
$$-1 = 10 - y$$

$$y = 11$$

$$\Rightarrow \quad C(8, 11)$$

* There are a number of alternative methods for part (c)

6. a)



i) $$ST = \sqrt{10^2 + 10^2}$$
$$= \sqrt{200}$$
$$= 10\sqrt{2} \text{ m}$$

ii) $$l = 10\sqrt{2} - 2x$$

$$\text{Area} = xl$$
$$= x(10\sqrt{2} - 2x)$$
$$= (10\sqrt{2})x - 2x^2$$

b) For maximum area, $A' = 0$

$$\Rightarrow \quad 10\sqrt{2} - 4x = 0$$
$$4x = 10\sqrt{2}$$
$$x = \frac{10\sqrt{2}}{4}$$
$$x = \frac{5\sqrt{2}}{2} \text{ m}$$

$$\Rightarrow \quad l = 10\sqrt{2} - 2\left(\frac{5\sqrt{2}}{2}\right)$$
$$= 10\sqrt{2} - 5\sqrt{2}$$
$$= 5\sqrt{2} \text{ m}$$

For maximum area, length $= 5\sqrt{2}$ m and breadth $= \frac{5\sqrt{2}}{2}$ m

7. $\int_0^2 \sin(4x+1)\,dx$

$= \left[-\frac{\cos(4x+1)}{4}\right]_0^2$

$= \left(-\frac{1}{4}\cos(9)\right) - \left(-\frac{1}{4}\cos(1)\right)$ * radian mode!

$= 0.363$ to 3 d.p.

8. when $x = a$, $y = 0$

$\log_3(a-1) - 2.2 = 0$

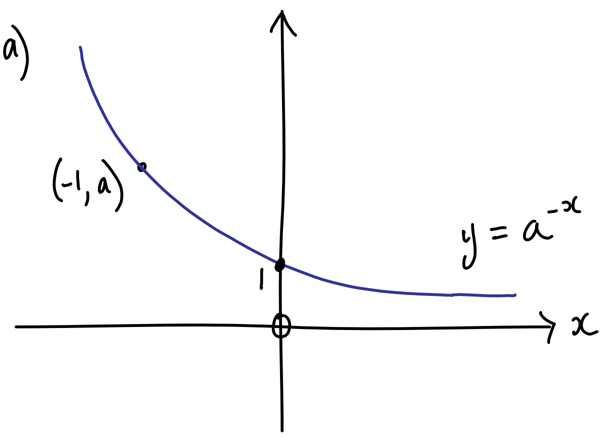
$\log_3(a-1) = 2.2$

$a - 1 = 3^{2.2}$

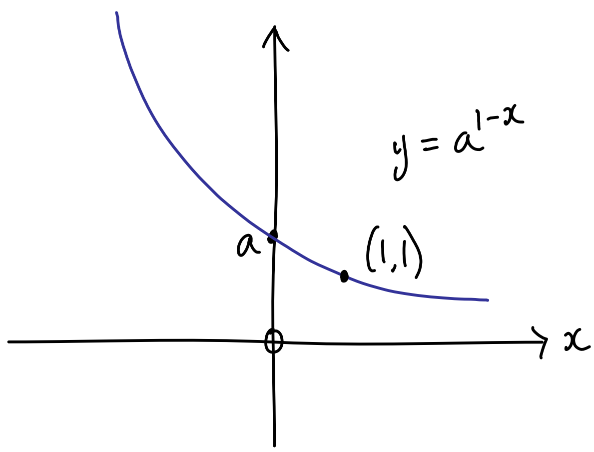
$a = 3^{2.2} + 1$

$a = 12.2$ to 1 d.p.

9. a)



b)



10. a) (i) $f'(x) = k(x-2)(x-4)$

$a = 2$, $b = 4$

(ii) $f'(0) = 6$

$\Rightarrow \quad k(-2)(-4) = 6$

$8k = 6$

$k = \frac{6}{8}$

$k = \frac{3}{4}$

b) $f'(x) = \frac{3}{4}(x-2)(x-4)$

$= \frac{3}{4}(x^2 - 6x + 8)$

$f(x) = \int \frac{3}{4}(x^2 - 6x + 8)\,dx$

$= \frac{3}{4}\left(\frac{x^3}{3} - 3x^2 + 8x\right) + C$

$= \frac{3x^3}{12} - \frac{9x^2}{4} + 6x + C$

$f(0) = 6$

$\Rightarrow \quad C = 6$

$\Rightarrow \quad f(x) = \frac{x^3}{4} - \frac{9x^2}{4} + 6x + 6$

11. a) $y = 3 \times 4^x$

when $x = a$, $y = 6$

$\Rightarrow \quad 6 = 3 \times 4^a$

$2 = 4^a$

$a = \frac{1}{2}$

b) when $x = -\frac{1}{2}$, $y = b$

$\Rightarrow \quad b = 3 \times 4^{-1/2}$

$b = 3 \cdot \frac{1}{\sqrt{4}}$

$b = \frac{3}{2}$

c) $y = 3 \times 4^x$

$\log_{10} y = \log_{10}(3 \times 4^x)$

$\log_{10} y = \log_{10} 3 + \log_{10} 4^x$

$\log_{10} y = x \log_{10} 4 + \log_{10} 3$

where $P = \log_{10} 4$ & $Q = \log_{10} 3$

$\Rightarrow \quad m = \log_{10} 4$