HIGHER 2008 PAPER 2 SOLUTIONS

1 (a) $A(7,a)$ $B(-3,-1)$ $C(5,-5)$

midpoint of BC, $M(1,-3)$

$$m_{BC} = \frac{-5-(-1)}{5-(-3)} = \frac{-4}{8}$$

$$m_{BC} = -\frac{1}{2}$$

$\Rightarrow$ $m_{AM} = 2$ as $m_1 m_2 = -1$ for $\perp$ lines

$$y - b = m(x - a)$$
$$y + 3 = 2(x - 1)$$
$$y + 3 = 2x - 2$$
$$y = 2x - 5$$

(b) Midpoint of AB, $N(2,4)$

$$m_{CN} = \frac{-5-4}{5-2} = \frac{-9}{3}$$

$$m_{CN} = -3$$

$$y - b = m(x - a)$$
$$y - 4 = -3(x - 2)$$
$$y - 4 = -3x + 6$$
$$y = -3x + 10$$

(c) for pts of intersection $y = y$

$\Rightarrow$
$$2x - 5 = -3x + 10$$
$$5x = 15$$
$$x = 3$$

when $x = 3$,
$$y = 2(3) - 5$$
$$y = 1$$

$\Rightarrow$ Point of intersection is $(3,1)$

2. (a) If P divides AE in ratio $2:1$

then z-coordinate of P will be $\frac{2}{3}$ z-coordinate of E

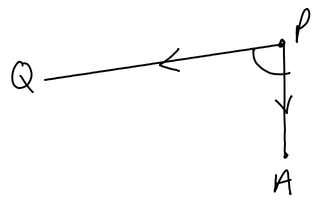
Hence $P(8, 0, 4)$

Similarly, the z-coordinate of Q will be $\frac{1}{2}$ z-coordinate of G

Hence $Q(0, 4, 3)$

(b) $\vec{PQ} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$ $\quad \vec{PA} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$ $\quad$ where $A(8, 0, 0)$

(c)



$|\vec{PQ}| = \sqrt{(-8)^2 + (4)^2 + (-1)^2}$

$= \sqrt{64 + 16 + 1}$

$= \sqrt{81}$

$= 9$

$|\vec{PA}| = \sqrt{(-4)^2}$

$= \sqrt{16}$

$= 4$

$\vec{PQ} \cdot \vec{PA} = |\vec{PQ}| . |\vec{PA}| \cos QPA$

$\cos QPA = \dfrac{\vec{PQ} \cdot \vec{PA}}{|\vec{PQ}||\vec{PA}|}$

$\cos QPA = \dfrac{(-8).0 + 4.0 + (-1)(-4)}{9.4}$

$\cos QPA = \dfrac{4}{36}$

$\angle QPA = \cos^{-1}\left(\frac{1}{9}\right)$

$\angle QPA = 83.6°$

3. (a) (i) $p = \sqrt{7}$ (ii) $q = -3$

(b) $f(x) + g(x) = \sqrt{7}\cos x - 3\sin x$

$$k\cos(x+a) = k\cos x\cos a - k\sin x \sin a$$

$k\cos a = \sqrt{7}$

$k\sin a = 3$

$k^2 = (\sqrt{7})^2 + (3)^2$

$k^2 = 7+9$

$k^2 = 16$

$k = 4$

$\dfrac{k\sin a}{k\cos a} = \tan a$

$\tan a = \dfrac{3}{\sqrt{7}}$

acute angle, $a = \tan^{-1}\left(\dfrac{3}{\sqrt{7}}\right)$

$a = 0.85$ rads

$$\left\{\begin{array}{l} a^\circ = 48.6^\circ \\ a = \dfrac{48.6\,\pi}{180} \\ a = 0.85 \text{ radians} \end{array}\right.$$

S ✓	A ✓✓✓
T ✓	C ✓

a is in 1st quadrant $\Rightarrow$ $a = 0.85$ rads

Hence $f(x) + g(x) = 4\cos(x + 0.85)$

(c) $f'(x) + g'(x) = -4\sin(x + 0.85)$

4. (a) $x^2 + y^2 + 8x + 4y - 38 = 0$

$C(-4,-2)$ $\quad r = \sqrt{(-4)^2 + (-2)^2 - (-38)}$

$r = \sqrt{16+4+38}$

$r = \sqrt{58}$

(b) $(x-4)^2 + (y-6)^2 = 26$

$C(4,6)$ $\quad r = \sqrt{26}$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(4-(-4))^2 + (6-(-2))^2}$

$d = \sqrt{(8)^2 + (8)^2}$

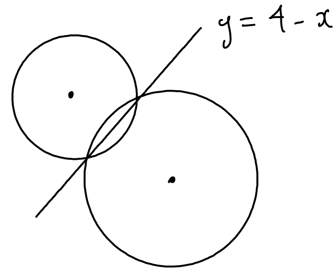
$d = \sqrt{128}$ $\quad$ distance between centres $= \sqrt{128}$

$= 11.3$ units

sum of radii $= \sqrt{58} + \sqrt{26}$

$= 12.7$ units

distance between centres is less than sum of radii
hence circles must intersect



(c) line intersects both circles at same points
so choose either circle:

for points of intersection $y = y_1$

$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$

$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1, 3$

when $x = -1$,

$y = 4 - (-1)$

$y = 5$

when $x = 3$

$y = 4 - 3$

$y = 1$

$\Rightarrow$ Points of intersection of the two circles are $(-1,5)$ and $(3,1)$

5.

$$\cos 2x^\circ + 2\sin x^\circ = \sin^2 x^\circ$$

$$1 - 2\sin^2 x^\circ + 2\sin x^\circ = \sin^2 x^\circ$$

$$3\sin^2 x - 2\sin x^\circ - 1 = 0$$

$$(3\sin x^\circ + 1)(\sin x^\circ - 1) = 0$$

$$\left\{\begin{array}{l} 3x^2 - 2x - 1 \\ (3x+1)(x-1) \end{array}\right.$$

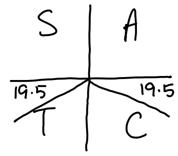
$$\Rightarrow \quad \sin x^\circ = -\frac{1}{3} \quad \text{or} \quad \sin x^\circ = 1$$

$$x^\circ = 90^\circ$$

$$x^\circ = \sin^{-1}\left(\frac{1}{3}\right)$$

$$(\text{acute angle } x^\circ = 19.5^\circ)$$

$$x^\circ = 199.5^\circ,\ 340.5^\circ$$



$$180^\circ + 19.5^\circ = 199.5^\circ$$

$$360 - 19.5^\circ = 340.5^\circ$$

$$\Rightarrow \quad x^\circ \in \{90^\circ, 199.5^\circ, 340.5^\circ\}$$

6. (a) gradient of line, $m = \frac{-6}{3}$

$$m = -2$$

$$\Rightarrow \quad y = -2x + 6$$

x-coordinate of R is t, so $y = -2t + 6$

ie $QR = 6 - 2t$

(b) Area of $OPQR = t(6 - 2t)$

$$A = 6t - 2t^2$$

for maximum area, $A' = 0$

$$\Rightarrow \quad 6 - 4t = 0$$

$$4t = 6$$

$$t = \frac{6}{4}$$

$$t = \frac{3}{2}$$

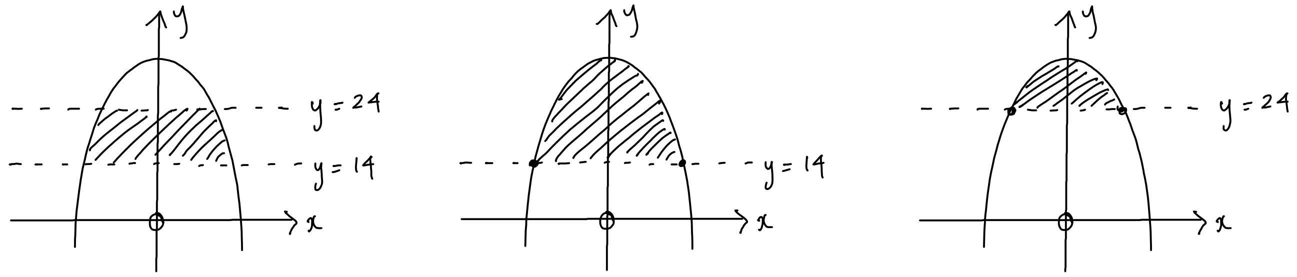
when $t = \frac{3}{2}$, $QR = 6 - 2\left(\frac{3}{2}\right)$

$$= 6 - 3$$

$$= 3$$

Hence $Q\left(\frac{3}{2}, 3\right)$

7.



$$A \quad = \quad A_1 \quad - \quad A_2$$

Need limits, ie pts of intersection of parabola and each line:

$$\begin{aligned} 32 - 2x^2 &= 14 \\ 2x^2 &= 18 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

$$\begin{aligned} 32 - 2x^2 &= 24 \\ 2x^2 &= 8 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

$$\begin{aligned} A_1 &= \int_{-3}^{3} \left(32 - 2x^2 - 14\right) dx \\ &= \int_{-3}^{3} \left(18 - 2x^2\right) dx \\ &= \left[18x - \frac{2x^3}{3}\right]_{-3}^{3} \\ &= \left(18(3) - \frac{2(3)^3}{3}\right) - \left(18(-3) - \frac{2(-3)^3}{3}\right) \\ &= (54 - 18) - (-54 + 18) \\ &= 36 - (-36) \\ &= 72 \end{aligned}$$

$$\begin{aligned} A_1 &= \int_{-2}^{2} \left(32 - 2x^2 - 24\right) dx \\ &= \int_{-2}^{2} \left(8 - 2x^2\right) dx \\ &= \left[8x - \frac{2x^3}{3}\right]_{-2}^{2} \\ &= \left(8(2) - \frac{2(2)^3}{3}\right) - \left(8(-2) - \frac{2(-2)^3}{3}\right) \\ &= \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right) \\ &= 32 - \frac{32}{3} \\ &= \frac{64}{3} \end{aligned}$$

$$\begin{aligned} \text{Shaded Area} &= 72 - \frac{64}{3} \\ &= \frac{216}{3} - \frac{64}{3} \\ &= \frac{152}{3} \text{ units}^2 \end{aligned}$$