2011 HIGHER PAPER 1 SOLUTIONS

1. $\underset{\sim}{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix} \quad \underset{\sim}{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \underset{\sim}{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$

$2\underset{\sim}{p} - \underset{\sim}{q} - \frac{1}{2}\underset{\sim}{r}$

$= 2\begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 10 \\ -14 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$

C

2. $3y + 2x = 6$

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

$m = -\frac{2}{3}$

B

3. $y = f(x+2) - 1$

shift 2 left, 1 down

$(1,2) \rightarrow (-1,1)$

$(-2,-3) \rightarrow (-4,-4)$

D

4. $y = x^3 - 2x \qquad (2,4)$

$\frac{dy}{dx} = 3x^2 - 2$

when $x = 2$

$m = 3(2)^2 - 2$

$m = 10$

D

5. $x^2 - 8x + 7$

$= (x-4)^2 - 16 + 7$

$= (x-4)^2 - 9$ (A)

6. $m_{cp} = -2$ $\quad P(2,-3)$

$m_T = \frac{1}{2}$ $\quad$ as $m_1 m_2 = -1$ for $\perp$ lines

$y - b = m(x - a)$

$y + 3 = \frac{1}{2}(x - 2)$ (C)

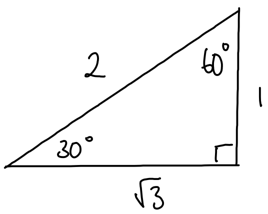
7. $x = 1$

$$\begin{array}{c|cccc} x=1 & 1 & -1 & 1 & 3 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & \boxed{4} \end{array}$$

(D)

8. $m = \tan 30°$

$m = \frac{1}{\sqrt{3}}$



(A)

9. $b^2 - 4ac = 23$

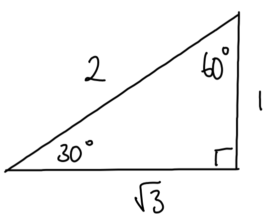
$b^2 - 4ac > 0$ so roots are real

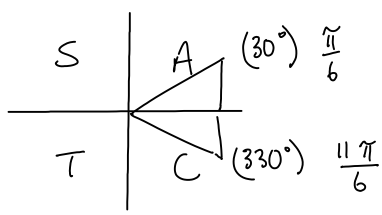
$\sqrt{23}$ is irrational so roots are irrational (B)

10. $2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$





(D)

11. $\int \left(4x^{1/2} + x^{-3}\right) dx$

$= \dfrac{4x^{3/2}}{3/2} + \dfrac{x^{-2}}{-2} + c$

$= \dfrac{2}{3} . 4x^{3/2} - \dfrac{1}{2}x^{-2} + c$

$= \dfrac{8}{3}x^{3/2} - \dfrac{1}{2}x^{-2} + c$ D

12. $\sin(p+q) = \sin p \cos q + \cos p \sin q$

$= \dfrac{2}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{3} + \dfrac{1}{\sqrt{5}} \cdot \dfrac{2}{3}$

$= \dfrac{2}{3} + \dfrac{2}{3\sqrt{5}}$ C

13. $f(x) = 4\sin 3x$

$f'(x) = 4\cos 3x . 3$

$= 12\cos 3x$

$f'(0) = 12\cos 3(0)$

$= 12$ C

14. $\underset{\sim}{p} \cdot \underset{\sim}{q} = |\underset{\sim}{p}| . |\underset{\sim}{q}| \cos\theta$

$= 3.3. \cos 60^\circ$

$= 9.\dfrac{1}{2}$

$= \dfrac{9}{2}$ B

15. $S(-4,5,1) \quad T(-16,-4,16) \quad U(-24,-10,26)$

$$\vec{ST} = \begin{pmatrix}-12\\-9\\15\end{pmatrix} \qquad \vec{TU} = \begin{pmatrix}-8\\-6\\10\end{pmatrix}$$

$$= 3\begin{pmatrix}-4\\-3\\5\end{pmatrix} \qquad = 2\begin{pmatrix}-4\\-3\\5\end{pmatrix}$$

$$ST : TU = 3:2$$

B

16. $$\int \frac{1}{3x^4}\,dx$$

$$= \int \tfrac{1}{3}x^{-4}\,dx$$

$$= \frac{1}{3}\,\frac{x^{-3}}{-3} + c$$

$$= \frac{x^{-3}}{-9} + c$$

A

$$= \frac{-1}{9x^3} + c$$

17. Roots at $x = -1, 0, 2$

$\Rightarrow$ factors $(x+1)$, x, $(x-2)$

$$\Rightarrow \quad y = k\,x(x+1)(x-2)$$

sub in $(1,2)$

$$2 = k.1.2.(-1)$$

$$2 = -2k$$

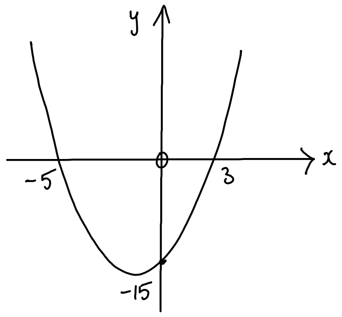
$$k = -1$$

$$\Rightarrow \quad y = -x(x+1)(x-2)$$

A

18. $f(x) = (x-3)(x+5)$

$= x^2 + 2x - 15$ $\qquad x^2$ +ve $\Rightarrow \cup$



(C)

$f(x) > 0$ when $x < -5$ and $x > 3$

19. $\log_3 y = x \iff y = 3^x$

(C)

20. $g(x) = \sin^2\sqrt{x-2}$

$\sqrt{x-2}$ cannot be negative so $x \geq 2$

$\sin^2\sqrt{x-2} = (\sin\sqrt{x-2})^2$ cannot be negative, max = 1, min = 0

$x \geq 2$, $0 \leq g(x) \leq 1$ (D)

21. (a) $B(7, 12)$ $D(2, -3)$

$$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3-12}{2-7}$$

$$= \frac{-15}{-5}$$

$$m_{BD} = 3$$

$$y - b = m(x-a)$$

$$y - 12 = 3(x-7)$$

$$y - 12 = 3x - 21$$

$$y = 3x - 9$$

(b) $x + 3y = 23$

$y = 3x - 9$

for pts of intersection $y = y$

$$x + 3(3x-9) = 23$$

$$x + 9x - 27 = 23$$

$$10x = 50$$

$$x = 5$$

when $x = 5$,

$$y = 3(5) - 9$$

$$y = 6$$

$E(5, 6)$

(c) (i) $A(-1, 8)$ $B(7, 12)$

$M(3, 10)$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12-8}{7-(-1)}$$

$$= \frac{4}{8}$$

$$= \frac{1}{2}$$

$m_\perp = -2$ as $m_1 m_2 = -1$ for $\perp$ lines

$$y - b = m(x-a)$$

$$y - 10 = -2(x-3)$$

$$y - 10 = -2x + 6$$

$$y = -2x + 16$$

(ii) when $x = 5$

$$y = -2(5) + 16$$

$$y = 6$$

$\Rightarrow$ line passes through E

22. (a) $f(x) = (x-2)(x^2+1)$

(i) on x-axis, $f(x) = 0$

$(x-2)(x^2+1) = 0$

$x = 2$ $\left(\begin{array}{c} x^2+1 = 0 \\ \text{no solutions} \end{array}\right)$

$(2,0)$

(ii) on y-axis $x = 0$

$f(0) = (-2).1$

$= -2$

$(0,-2)$

(b) $f(x) = (x-2)(x^2+1)$

$= x^3 - 2x^2 + x - 2$

$f'(x) = 3x^2 - 4x + 1$

for stat pts $f'(x) = 0$

$3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}, 1$

$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right) - 2$

$= \frac{1}{27} - \frac{2}{9} + \frac{1}{3} - 2$

$= \frac{1}{27} - \frac{6}{27} + \frac{9}{27} - \frac{54}{27}$

$= \frac{-50}{27}$

$f(1) = (1)^3 - 2(1)^2 + (1) - 2$

$= -2$

stat pts at $\left(\frac{1}{3}, \frac{-50}{27}\right)$ & $(1,-2)$

x	$\rightarrow$	$\frac{1}{3}$	$\rightarrow$	1	$\rightarrow$
$f'(x)$	$+$	0	$-$	0	$+$
shape	/	—	\	—	/

$f'(0) = 3(0)^2 - 4(0) + 1$

$= 1$

$f'\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1$

$= \frac{3}{4} - 2 + 1$

$= -\frac{1}{4}$

$f'(2) = 3(2)^2 - 4(2) + 1$

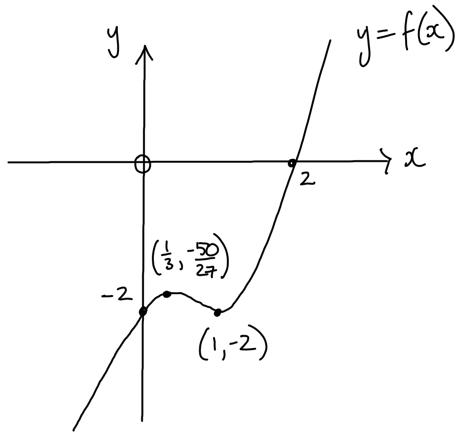
$= 12 - 8 + 1$

$= 5$

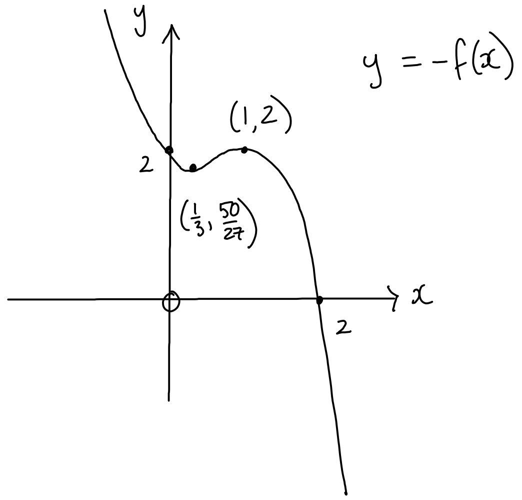
Max T.P. at $\left(\frac{1}{3}, -\frac{50}{27}\right)$

Min T.P at $(1,-2)$

(c) (i)



(ii)



23. (a) $\cos 2x^\circ - 3\cos x^\circ + 2 = 0 \qquad 0 \leq x < 360$

$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$

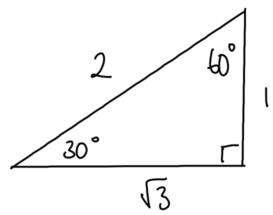
$2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0 \qquad\qquad 2x^2 - 3x + 1$

$(2\cos x^\circ - 1)(\cos x^\circ - 1) = 0 \qquad\qquad (2x-1)(x-1)$

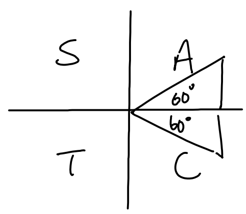
either $2\cos x^\circ - 1 = 0$ or $\cos x^\circ - 1 = 0$

$\cos x^\circ = \frac{1}{2} \qquad \cos x^\circ = 1$

$x^\circ = 60^\circ, 300^\circ \qquad x^\circ = 0^\circ, 360^\circ$



$x^\circ \in \{0^\circ, 60^\circ, 300^\circ\}$ (discard 360° as outwith domain)



(b) $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$

$\cos 2(2x)^\circ - 3\cos(2x)^\circ + 2 = 0$

from (a) $\quad 2x^\circ = 60^\circ, 300^\circ \qquad 2x^\circ = 0^\circ, 360^\circ$

$x^\circ = 30^\circ, 150^\circ, 210^\circ, 330^\circ \qquad x^\circ = 0^\circ, 180^\circ, 360^\circ$

$x^\circ \in \{0^\circ, 30^\circ, 150^\circ, 180^\circ, 210^\circ, 330^\circ\}$