HIGHER MATHS 2012 PAPER 1

1.

$u_{n+1} = 3u_n + 4 \quad , \quad u_0 = 1$

$u_1 = 3(1) + 4$
$\quad = 7$

C

$u_2 = 3(7) + 4$
$\quad = 25$

2.

$y = x^3 - 6x + 1$

$\frac{dy}{dx} = 3x^2 - 6$

when $x = -2$,

D

$\frac{dy}{dx} = 3(-2)^2 - 6$

$\quad = 3.4 - 6$

$m = 6$

3.

$x^2 - 6x + 14$

$= (x-3)^2 - 9 + 14$

B

$= (x-3)^2 + 5$

$q = 5$

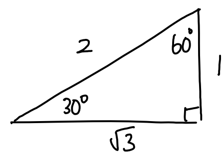
4.

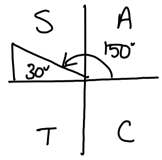
$m = \tan 150^\circ$

$m = -\tan 30^\circ$

$m = -\frac{1}{\sqrt{3}}$

B





5.

$\cos a = \frac{4}{5}$

$\cos 2a = 2\cos^2 a - 1$

$= 2\left(\frac{4}{5}\right)^2 - 1$

$= 2 . \frac{16}{25} - 1$

$= \frac{32}{25} - \frac{25}{25}$

A

$= \frac{7}{25}$

($\cos 2a = 1 - 2\sin^2 a$
$\quad = \cos^2 a - \sin^2 a$ can also be used)

6.

$y = 3x^{-2} + 2x^{3/2}$

$\frac{dy}{dx} = -6x^{-3} + \frac{3}{2} . 2x^{1/2}$

$= -6x^{-3} + 3x^{1/2}$

C

7.

If $\underline{u}$ & $\underline{v}$ are $\perp$ then $\underline{u} . \underline{v} = 0$

$(-3).1 + 1.t + 2t.(-1) = 0$

$-3 + t - 2t = 0$

A

$-t = 3$

$t = -3$

8. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 3 \cdot \frac{4}{3}\pi r^2$

$= 4\pi r^2$

C

when $r = 2$, $\frac{dV}{dr} = 4\pi(2)^2$

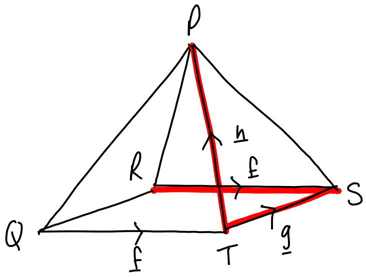
$= 16\pi$

9. Graph shows $y = \cos x$ shifted right by $\frac{\pi}{6}$ and down 1

$\Rightarrow \quad y = \cos\left(x - \frac{\pi}{6}\right) - 1$

A

10.



B

$\overrightarrow{RP} = -\underline{f} - \underline{g} + \underline{h}$

11. $\int \left(\frac{1}{6x^2}\right) dx$

$= \int \frac{1}{6} x^{-2}\, dx$

$= \frac{1}{6}\frac{x^{-1}}{-1} + C$

$= -\frac{1}{6}x^{-1} + C$

D

12.

reflected on x-axis

$2 - 3\sin\left(x - \frac{\pi}{3}\right)$

moved up 2

moved right $\frac{\pi}{3}$ (60°)

multiplied by 3

$\sin x$ has max value of 1

$\Rightarrow \quad 2 - 3(1) = -1$

$\Rightarrow \quad$ min value $= -1$

$\sin x$ has min value of -1

$\Rightarrow \quad 2 - 3(-1) = 5$

new max value $= 5$

$\sin x$ has max at $x = \frac{\pi}{2}$ $\quad \Rightarrow \quad$ Max value $= 5$ when $x = \frac{11\pi}{6}$

__But__ function is inverted so

max now at $\frac{3\pi}{2}$ shifted right $\frac{\pi}{3}$ $\Rightarrow \frac{11\pi}{6}$

$(270° + 60° = 330°)$

B

13. Roots at $x = -2$ and -1 $\Rightarrow$

$y = k(x+1)(x+2)$

$y = k(x^2 + 3x + 2)$

$y = kx^2 + 3kx + 2k$

y-intercept

$\Rightarrow 2k = 6$

$k = 3$

$y = 3(x+1)(x+2)$

D

14. $\int (2x-1)^{1/2}\, dx$

$= \dfrac{(2x-1)^{3/2}}{\frac{3}{\cancel{2}}\cdot\cancel{2}} + c$

$= \frac{1}{3}(2x-1)^{3/2} + c$

A

15. $\underline{u} = k\begin{pmatrix}3\\-1\\0\end{pmatrix}$

If $\underline{u}$ is a unit vector then $|\underline{u}| = 1$

$\sqrt{(3k)^2 + (-k)^2 + 0^2} = 1$

$10k^2 = 1$

$\sqrt{10}\,k = 1 \quad (k > 0)$

$k = \frac{1}{\sqrt{10}}$

D

16. $y = 3\cos^4 x$

$= 3(\cos x)^4$

(Chain Rule)

$\frac{dy}{dx} = 12(\cos x)^3 \cdot (-\sin x)$

$= -12\cos^3 x \sin x$

C

17. $\underline{a} = \begin{pmatrix}3\\4\\0\end{pmatrix}$

$|\underline{a}| = \sqrt{3^2+4^2+0^2}$

$= \sqrt{25}$

$= 5$

$\underline{a}.(\underline{a}+\underline{b}) = 7$

$\underline{a}.\underline{a} + \underline{a}.\underline{b} = 7$

$5.5 + \underline{a}.\underline{b} = 7$

$\underline{a}.\underline{b} = 7 - 25$

$\underline{a}.\underline{b} = -18$

D

18. (1) $f(x) < 0$ for $s < x < t$ TRUE

(function below x-axis between s and t)

B

(2) $f'(x) < 0$ for $x < q$ FALSE

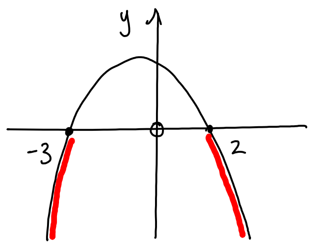
(gradient of $f(x)$ is negative for all x less than q)

gradient at $x = 0$ is 0 due to stat point (point of inflexion)

19. $6 - x - x^2 < 0$

$(3+x)(2-x) < 0$

roots are $x = -3, 2$



B

$x < -3 \qquad x > 2$

20. $\dfrac{\log_b 9a^2}{\log_b 3a}$

$= \dfrac{\log_b (3a)^2}{\log_b 3a}$

$= \dfrac{2\cancel{\log_b 3a}^{1}}{\cancel{\log_b 3a}^{1}}$

$= 2$

A

21. (a) (i)

$$\begin{array}{r|rrrr} x=4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array} \Rightarrow (x-4) \text{ is a factor}$$

Quotient: $x^2 - x - 2$

(ii)

$$x^3 - 5x^2 + 2x + 8$$
$$= (x-4)(x^2 - x - 2)$$
$$= (x-4)(x-2)(x+1)$$

(iii)

$$x^3 - 5x^2 + 2x + 8 = 0$$
$$x = -1, 2, 4$$

(b)

$$\int_0^2 (x^3 - 5x^2 + 2x + 8)\, dx$$
$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x \right]_0^2$$
$$= \left(\frac{(2)^4}{4} - \frac{5(2)^3}{3} + (2)^2 + 8(2) \right) - 0$$
$$= \frac{16}{4} - \frac{40}{3} + 4 + 16$$
$$= 24 - \frac{40}{3}$$
$$= \frac{72}{3} - \frac{40}{3}$$
$$= \frac{32}{3}$$

Shaded area $= \frac{32}{3}$ units^2

22. (a) $\cos x - \sqrt{3}\sin x$

$$k\cos(x+a) = k\cos x\cos a - k\sin x\sin a$$

$k\cos a = 1$

$-k\sin a = -\sqrt{3}$

$k\sin a = \sqrt{3}$

$k^2 = 1^2 + (\sqrt{3})^2$

$k^2 = 4$

$k = 2$

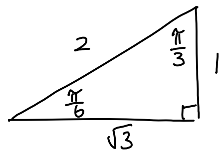
$\tan a = \dfrac{k\sin a}{k\cos a}$

$\tan a = \dfrac{\sqrt{3}}{1}$

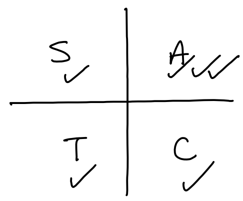
Acute angle $a = \dfrac{\pi}{3}$

a is in 1st quadrant

$\Rightarrow a = \dfrac{\pi}{3}$



$\tan \dfrac{\pi}{3} = \sqrt{3}$



$$\cos x - \sqrt{3}\sin x = 2\cos\left(x + \frac{\pi}{3}\right)$$

$$k = 2, \quad a = \frac{\pi}{3}$$

(b) On y-axis, $x = 0$

$\Rightarrow y = 2\cos\left(\dfrac{\pi}{3}\right)$

$y = 1$

$(0, 1)$

On x-axis, $y = 0$

$\Rightarrow 2\cos\left(x + \dfrac{\pi}{3}\right) = 0$

$\cos\left(x + \dfrac{\pi}{3}\right) = 0$

$x + \dfrac{\pi}{3} = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

$x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$

$\left(\dfrac{\pi}{6}, 0\right) \quad \left(\dfrac{7\pi}{6}, 0\right)$

23. (a) $P(3,-3) \quad Q(-1,9)$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{2}{2}, \frac{6}{2}\right)$$

$$M = (1,3)$$

$$m_{PQ} = \frac{y_2-y_1}{x_2-x_1}$$

$$m_{PQ} = \frac{-3-9}{3-(-1)}$$

$$m_{PQ} = \frac{-12}{4}$$

$$m_{PQ} = -3$$

$$m_{\perp} = \frac{1}{3}$$

as $m_1 m_2 = -1$ for $\perp$ lines

$$y-b = m(x-a)$$

$$y-3 = \tfrac{1}{3}(x-1)$$

$$3y-9 = x-1$$

$$3y = x+8$$

(b)

$$y-b=m(x-a)$$

$$y+2 = -3(x-1)$$

$$y+2 = -3x+3$$

$$y = -3x+1$$

(c) For points of intersection $y=y$

$$3(-3x+1) = x+8$$

$$-9x+3 = x+8$$

$$-10x = 5$$

$$x = -\tfrac{1}{2}$$

when $x = -\frac{1}{2}$

$$y = -3\left(-\tfrac{1}{2}\right)+1$$

$$y = \tfrac{3}{2}+1$$

$$y = \tfrac{5}{2}$$

Point of intersection $\left(-\frac{1}{2}, \frac{5}{2}\right)$

(d) Shortest distance is from midpoint to point of intersection of l_1 and l_2

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d = \sqrt{\left(1-\left(-\tfrac{1}{2}\right)\right)^2 + \left(3-\tfrac{5}{2}\right)^2}$$

$$d = \sqrt{\left(\tfrac{3}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2}$$

$$d = \sqrt{\left(\tfrac{9}{4}+\tfrac{1}{4}\right)}$$

$$d = \sqrt{\tfrac{10}{4}}$$

$$d = \frac{\sqrt{10}}{2} \text{ units}$$