HIGHER MATHS 2012 PAPER 2

1. (a) $f(x) = x^2 + 3$ $g(x) = x + 4$

(i) $f(g(x)) = f(x+4)$

$= (x+4)^2 + 3$

(ii) $g(f(x)) = g(x^2+3)$

$= (x^2+3) + 4$

$= x^2 + 7$

(b) $f(g(x)) + g(f(x)) = 0$

$(x+4)^2 + 3 + x^2 + 7 = 0$

$x^2 + 8x + 16 + 3 + x^2 + 7 = 0$

$2x^2 + 8x + 26 = 0$

$x^2 + 4x + 13 = 0$

$b^2 - 4ac = (4)^2 - 4(1)(13)$

$= 16 - 52$

$= -36$

$b^2 - 4ac < 0 \Rightarrow$ no real roots

2. (a) $2x - y + 5 = 0$ $\qquad$ $x^2 + y^2 - 6x - 2y - 30 = 0$

$y = 2x + 5$

for points of intersection $y = y$

$$x^2 + (2x+5)^2 - 6x - 2(2x+5) - 30 = 0$$
$$x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$$
$$5x^2 + 10x - 15 = 0$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x = -3, 1$$

when $x = -3$

$y = 2(-3) + 5$
$y = -1$
$P(-3,-1)$

when $x = 1$

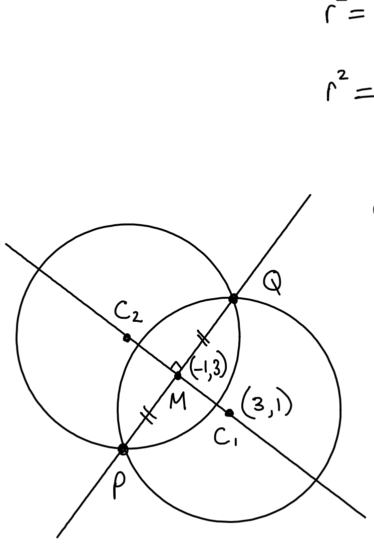
$y = 2(1) + 5$
$y = 7$
$Q(1,7)$

(b) $x^2 + y^2 - 6x - 2y - 30 = 0$

$C_1(3,1)$ $\qquad$ $r^2 = (3)^2 + (1)^2 - (-30)$

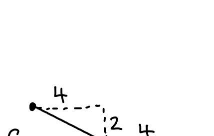
$r^2 = 9 + 1 + 30$

$r^2 = 40$



$C_1(3,1)$

$M(-1,3)$



$C_2(-5,5)$

Second circle : $(x+5)^2 + (y-5)^2 = 40$

3. $f(x) = x^3 - 2x^2 - 4x + 6 \qquad 0 \leq x \leq 3$

$f'(x) = 3x^2 - 4x - 4$

for stat. pts $f'(x) = 0$

$3x^2 - 4x - 4 = 0$

$$\begin{array}{cc} (3 & 2) \\ (1 & -2) \\ \hline 2 & -6 \end{array}$$

$(3x+2)(x-2) = 0$

$3x + 2 = 0$ or $x - 2 = 0$

$x = -\frac{2}{3} \qquad x = 2$

$f'(-1) = 3(-1)^2 - 4(-1) - 4$
$= 3 + 4 - 4$
$= 3$

$f'(0) = -4$

$f'(3) = 3(3)^2 - 4(3) - 4$
$= 27 - 12 - 4$
$= 11$

x	$\rightarrow$	$-\frac{2}{3}$	$\rightarrow$	2	$\rightarrow$
$f'(x)$	$+$	0	$-$	0	$+$
Shape	/	—	\	—	/

$f(2) = (2)^3 - 2(2)^2 - 4(2) + 6$
$= -2$

Max T.P out with domain

Min T.P. at $(2,-2)$

Check endpoints:

$f(0) = 6$

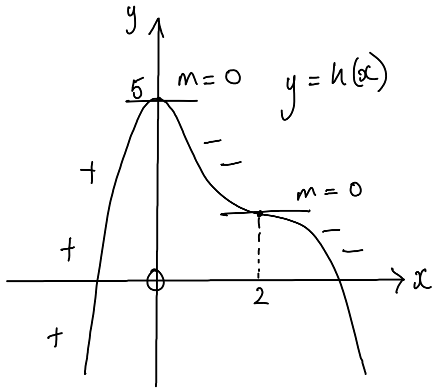
$(0,6)$

$f(3) = (3)^3 - 2(3)^2 - 4(3) + 6$
$= 3$

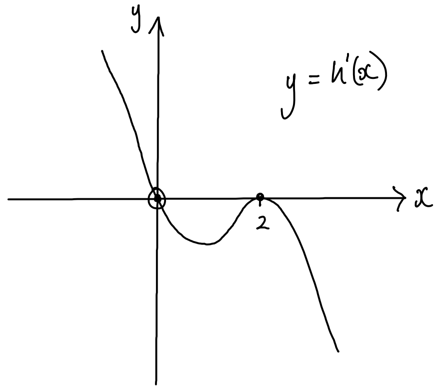
$(3,3)$

Maximum value of f is 6, minimum value of f is -2.

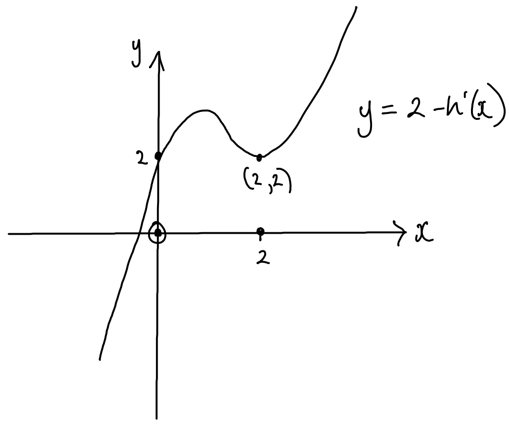
4.



(a)



(b)



5. (a) $A(3,-3,0)$ $B(2,-3,1)$ $C(4,k,0)$

(i) $\vec{BA} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} 2 \\ k+3 \\ -1 \end{pmatrix}$

(ii) $\vec{BA} \cdot \vec{BC} = 1.2 + 0.(k+3) + (-1)(-1)$

$= 2 + 0 + 1$

$= 3$

$|\vec{BA}| = \sqrt{(1)^2 + (0)^2 + (-1)^2}$

$= \sqrt{2}$

$|\vec{BC}| = \sqrt{(2)^2 + (k+3)^2 + (-1)^2}$

$= \sqrt{5 + k^2 + 6k + 9}$

$= \sqrt{k^2 + 6k + 14}$

$\cos A\hat{B}C = \dfrac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}||\vec{BC}|}$

$\cos A\hat{B}C = \dfrac{3}{\sqrt{2}\left(\sqrt{k^2 + 6k + 14}\right)}$

$\cos A\hat{B}C = \dfrac{3}{\sqrt{2(k^2+6k+14)}}$

(b) $\cos 30^\circ = \dfrac{3}{\sqrt{2(k^2+6k+14)}}$

$\dfrac{\sqrt{3}}{2} = \dfrac{3}{\sqrt{2(k^2+6k+14)}}$

$\dfrac{3}{4} = \dfrac{9}{2(k^2+6k+14)}$

$6(k^2+6k+14) = 36$

$k^2 + 6k + 14 = 6$

$k^2 + 6k + 8 = 0$

$(k+4)(k+2) = 0$

$k = -4, -2$

6. (a) $u_{n+1} = (\sin x)u_n + \cos 2x, \quad u_0 = 1$

for $0 < x < \frac{\pi}{2}$, $\quad 0 < \sin x < 1 \quad \Rightarrow$ sequence has a limit

(b) At the limit $u_{n+1} = u_n = L$

$$L = L\sin x + \cos 2x$$

$$L - L\sin x = \cos 2x$$

$$L(1-\sin x) = \cos 2x$$

$$L = \frac{\cos 2x}{1-\sin x}$$

OR

$$L = \frac{c}{1-m}$$

$$L = \frac{\cos 2x}{1-\sin x}$$

$$\tfrac{1}{2}\sin x = \frac{\cos 2x}{1-\sin x}$$

$$\tfrac{1}{2}\sin x - \tfrac{1}{2}\sin^2 x = \cos 2x$$

$$\tfrac{1}{2}\sin x - \tfrac{1}{2}\sin^2 x = 1 - 2\sin^2 x$$

$$\tfrac{3}{2}\sin^2 x + \tfrac{1}{2}\sin x - 1 = 0$$

$$\tfrac{1}{2}(3\sin^2 x + \sin x - 2) = 0$$

$$\tfrac{1}{2}(3\sin x - 2)(\sin x + 1) = 0$$

either $3\sin x - 2 = 0$ or $\sin x + 1 = 0$

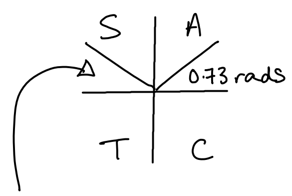
$\sin x = \frac{2}{3}$ $\qquad$ $\sin x = -1$

Acute angle $= 0.73$ rads $\qquad$ $x = \frac{3\pi}{2}$ (4.71 rads)

$x = 0.73, 2.41$ rads

$\sin^{-1}\left(\frac{2}{3}\right) = 41.8^\circ$

$= 0.73$ rads



$\pi - 0.73 = 2.41$ rads

$x = 0.73, 2.41, 4.71$ radians

Given domain $0 < x < \frac{\pi}{2}$ for a limit $\underline{\underline{x = 0.73}}$

7. (a) For points of intersection $y = y$

$$3^{2-x} = 4^x$$

$$\log 3^{2-x} = \log 4^x$$

$$(2-x)\log 3 = x \log 4$$

$$\frac{2-x}{x} = \frac{\log 4}{\log 3}$$

$$\frac{2}{x} - 1 = \frac{\log 4}{\log 3}$$

$$\frac{2}{x} = \frac{\log 4}{\log 3} + \frac{\log 3}{\log 3}$$

$$\frac{2}{x} = \frac{\log 4 + \log 3}{\log 3}$$

$$\frac{2}{x} = \frac{\log 12}{\log 3}$$

$$\frac{x}{2} = \frac{\log 3}{\log 12}$$

$$x = \frac{\log 3^2}{\log 12}$$

$$x = \frac{\log 9}{\log 12}$$

(b)

$$x = \frac{\log_{10} 9}{\log_{10} 12} \qquad \left(\frac{\log_e 9}{\log_e 12} = 0.88 \right)$$

$$x = 0.88$$

$$y = 4^{0.88}$$

$$y = 3.39 \qquad T(0.88, 3.39)$$