1. $u_1 = 4 \quad u_2 = 7 \quad u_3 = 16$

$u_{n+1} = mu_n + c$

$u_2 = mu_1 + c$

$7 = 4m + c$

$c = 7 - 4m$

$u_3 = mu_2 + c$

$16 = 7m + c$

$c = 16 - 7m$

$\Rightarrow \quad 7 - 4m = 16 - 7m$

$3m = 9$

$\underline{\underline{m = 3}}$

when $m = 3$, $c = 7 - 4(3)$

$\underline{\underline{c = -5}}$

$u_{n+1} = 3u_n - 5$

2. (a) $QR \perp QP$

$Q(5,6) \quad P(7,2)$

$m_{QP} = \frac{2-6}{7-5}$

$m_{QP} = \frac{-4}{2}$

$m_{QP} = -2$

$\Rightarrow m_{QR} = \frac{1}{2}$ as $m_1 m_2 = -1$ for $\perp$ lines

$Q(5,6) \quad m = \frac{1}{2}$

$y - b = m(x - a)$

$y - 6 = \frac{1}{2}(x - 5)$

$2y - 12 = x - 5$

$x - 2y + 7 = 0$

(b) QR: $x = 2y - 7$

PT: $x = 13 - 3y$

for points of intersection $x = x$

$\Rightarrow \quad 2y - 7 = 13 - 3y$

$5y = 20$

$y = 4$

when $y = 4$, $x = 2(4) - 7$

$x = 1$

$T(1,4)$

(c) $Q(5,6) \quad T(1,4) \quad P(7,2)$

$\vec{TR} = \vec{QT} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

$\Rightarrow R(-3,2)$

$\vec{RS} = \vec{QP} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$\Rightarrow S(-1,-2)$

3. (a) $x^3 + 3x^2 + x - 5$

$$\begin{array}{r|rrrr} x = 1 & 1 & 3 & 1 & -5 \\ & & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & \boxed{0} \end{array} \Rightarrow (x-1) \text{ is a factor}$$

Quotient is $x^2 + 4x + 5$

check discriminant: $b^2 - 4ac$
$= (4)^2 - 4(1)(5)$
$= 16 - 20$
$= -4$

$b^2 - 4ac < 0 \Rightarrow$ quotient won't factorise

not necessarily worth marks

$x^3 + 3x^2 + x - 5$

$= (x-1)(x^2 + 4x + 5)$

(b) $y = x^4 + 4x^3 + 2x^2 - 20x + 3$

$\frac{dy}{dx} = 4x^3 + 12x^2 + 4x - 20$

for stat. pts. $\frac{dy}{dx} = 0$

$4x^3 + 12x^2 + 4x - 20 = 0$

$x^3 + 3x^2 + x - 5 = 0$

$(x-1)(x^2 + 4x + 5) = 0$ from (a)

$x = 1$

only one stat pt as $x^2 + 4x + 5$ has no solutions

x	$\rightarrow$	1	$\rightarrow$
$\frac{dy}{dx}$	$-$	0	$+$
shape	\	—	/

when $x = 0$
$\frac{dy}{dx} = -5$

when $x = 2$
$\frac{dy}{dx} = (2)^3 + 3(2)^2 + (2) - 5$
$= 8 + 12 + 2 - 5$
$= 17$

Min T.P. at $x = 1$

4. for points of intersection $y = y$

$$x^3 + 3x^2 + 2x + 3 = 2x + 3$$
$$x^3 + 3x^2 = 0$$
$$x^2(x+3) = 0$$
$$x = -3, 0$$

A is at $x = 0$

when $x = -3$,

$$y = 2(-3) + 3$$
$$y = -3$$

Hence $B(-3,-3)$

$$\int_{-3}^{0} \left(x^3 + 3x^2 + 2x + 3 - (2x+3)\right) dx$$
$$= \int_{-3}^{0} (x^3 + 3x^2)\, dx$$
$$= \left[\frac{x^4}{4} + x^3\right]_{-3}^{0}$$
$$= 0 - \left(\frac{(-3)^4}{4} + (-3)^3\right)$$
$$= -\left(\frac{81}{4} - 27\right)$$
$$= 27 - \frac{81}{4}$$
$$= \frac{108}{4} - \frac{81}{4}$$
$$= \frac{27}{4} \text{ units}^2$$

5.
$$\log_5(3-2x) + \log_5(2+x) = 1$$
$$\log_5((3-2x)(2+x)) = 1 \qquad (\text{using } \log a + \log b = \log ab)$$
$$5^1 = (3-2x)(2+x)$$
$$5 = 6 + 3x - 4x - 2x^2$$
$$2x^2 + x - 1 = 0$$
$$(2x-1)(x+1) = 0$$
$$x = -1, \tfrac{1}{2}$$

6.
$$\int_0^a 5\sin 3x\, dx = \frac{10}{3} \qquad 0 \leqslant a < \pi$$
$$\left[\frac{-5\cos 3x}{3}\right]_0^a = \frac{10}{3}$$
$$-\frac{5}{3}\cos 3a - \left(-\frac{5}{3}\cos 0\right) = \frac{10}{3}$$
$$-\frac{5}{3}\cos 3a + \frac{5}{3} = \frac{10}{3}$$
$$-\frac{5}{3}\cos 3a = \frac{5}{3}$$
$$\cos 3a = -1$$
$$3a = \pi$$
$$a = \frac{\pi}{3}$$

7. (a)

$$L = 3x + 4y \qquad A = 2xy$$

$$L = 3x + 4 \cdot \frac{12}{x} \qquad 24 = 2xy$$

$$y = \frac{24}{2x}$$

$$L = 3x + \frac{48}{x} \qquad y = \frac{12}{x}$$

(b) (i) For minimum length, $L' = 0$

$$L = 3x + 48x^{-1}$$

$$L' = 3 - 48x^{-2}$$

$$L' = 3 - \frac{48}{x^2}$$

$$\Rightarrow \quad 3 - \frac{48}{x^2} = 0$$

$$3x^2 - 48 = 0$$

$$3x^2 = 48 \qquad \underline{\text{OR}} \qquad 3(x^2 - 16) = 0$$

$$x = 16 \qquad 3(x+4)(x-4) = 0$$

$$x = \pm 4 \qquad x = -4, 4$$

$$\Rightarrow \quad x = 4\text{m}$$

(ii) when $x = 4$

$$L = 3(4) + \frac{48}{(4)}$$

$$L = 24\text{m}$$

Cost = 24 × £8·25

= £198

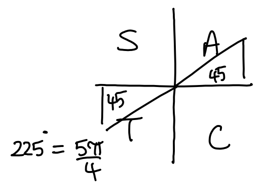
8.

$$\sin 2x = 2\cos^2 x \qquad 0 \leqslant x < 2\pi$$

$$2\sin x \cos x = 2\cos^2 x$$

$$2\sin x \cos x - 2\cos^2 x = 0$$

$$2\cos x(\sin x - \cos x) = 0$$



$$2\cos x = 0 \qquad \text{or} \qquad \sin x - \cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin x = \cos x$$

either remember $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ or divide by $\cos x$

$$x = \frac{\pi}{4} \qquad \underline{\text{OR}} \qquad \frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

solution: $x \in \left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}\right\}$

9. (a) $P_t = P_0 e^{-kt}$

when $t = 25$, $P_{25} = 0.5$, $P_0 = 1$

$0.5 = e^{-25k}$

$\log_e 0.5 = \log_e e^{-25k}$

$\log_e 0.5 = -25k \log_e e$ $\quad (\log_e e = 1)$

$-25k = \log_e 0.5$

$k = \dfrac{\log_e 0.5}{-25}$

$k = 0.028$ to 2 sig.fig.

(b) $P_t = P_0 e^{-0.028t}$

start with 100%

when $t = 80$

$P = 100 e^{-0.028(80)}$

$P = 100 e^{-2.24}$

$P = 10.6$

10.6% remaining so $\underline{\underline{89.4\%}}$ decrease.