HIGHER MATHS 2014 PAPER 1 SOLUTIONS

1. $u_{n+1} = \frac{1}{3}u_n + 1 \qquad u_2 = 15$

$u_3 = \frac{1}{3}(15) + 1$
$= 6$

$u_4 = \frac{1}{3}(6) + 1$
$= 3$ (C)

2. $C(1,2) \quad T(3,-1)$

$m_r = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad m_T = \frac{2}{3}$ as $m_1 m_2 = -1$ for $\perp$ lines

$m_r = \frac{-1-2}{3-1}$

$m_r = -\frac{3}{2}$ (B)

3. $\log_4 12 - \log_4 x = \log_4 6$

$\log_4 \frac{12}{x} = \log_4 6$

$\frac{12}{x} = 6$

$x = 2$ (A)

4. $3\sin x - 4\cos x$

$k\cos(x-a) = k\cos x \cos a + k\sin x \sin a$

$k\sin a = 3$

$k\cos a = -4$ (D)

5. $\int (2x+9)^5 \, dx$

$= \frac{(2x+9)^6}{6 \cdot 2} + c$

$= \frac{1}{12}(2x+9)^6 + c$ (D)

6. $\underline{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \quad \underline{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

$$2\underline{u} - 3\underline{v} = 2\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$$

(A)

7. $\sin a = \frac{3}{\sqrt{34}} \qquad \cos a = \frac{5}{\sqrt{34}}$

$$\sin 2a = 2\sin a \cos a$$

$$= 2 \cdot \frac{3}{\sqrt{34}} \cdot \frac{5}{\sqrt{34}}$$

$$= \frac{30}{34}$$

$$= \frac{15}{17}$$

(C)

8. $\frac{d}{dx}\left(4 - 9x^4\right)^{1/2}$

$$= \frac{1}{2}\left(4 - 9x^4\right)^{-1/2} \cdot -36x^3$$

$$= -18x^3\left(4 - 9x^4\right)^{-1/2}$$

(D)

9. $5\sin 2x + 5\sqrt{3}\cos 2x$

$= 5(\sin 2x + \sqrt{3}\cos 2x)$

max value = 10

$\sin x + \sqrt{3}\cos x$ has max. value of 2

$5(\sin x + \sqrt{3}\cos x)$ has max. value of 10

↑ affects no. of waves in 360° not max/min

(B)

10. $u_{n+1} = (k-2)u_n + 5 \qquad u_0 = 3$

For a limit to exist $\quad -1 < (k-2) < 1$

$$1 < k < 3$$

(C)

11. $y = 2f(x) + 1$ ← move up 1

↑ double y coordinates

$(2, 3) \rightarrow (2, 7)$

$(5, 0) \rightarrow (5, 1)$ (C)

12. $f(x) = \dfrac{6x}{x^2 + 6x - 16}$

cannot divide by zero

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

$x = -8, 2$

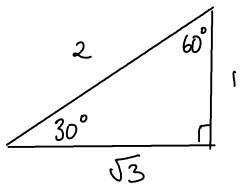
$\Rightarrow \quad x \neq -8 \quad \text{or} \quad x \neq 2$

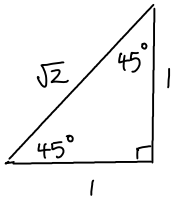
(A)

13. $\sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{4}\right)$

$= \frac{\sqrt{3}}{2} - \left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$

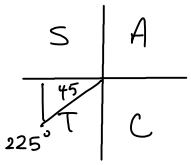




(B)

$\sin\left(\frac{\pi}{3}\right) = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{5\pi}{4}\right) = \cos 225° = -\cos 45° = -\frac{1}{\sqrt{2}}$



14. $\underline{u} = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix}$ $\underline{v} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$

if $\underline{u}$ & $\underline{v}$ $\perp$ then $\underline{u}.\underline{v} = 0$

$\Rightarrow$ $1.(-6) + k.2 + k.5 = 0$

$-6 + 7k = 0$

$7k = 6$

$k = \frac{6}{7}$ (D)

15. Root at $x = -1$ and repeated root at $x = 2$

$\Rightarrow$ $y = a(x+1)(x-2)^2$

y-intercept at $y = -8$ $(0, -8)$

$\Rightarrow$ $-8 = a(0+1)(0-2)^2$

$-8 = 4a$

$a = -2$

$\Rightarrow$ $y = -2(x+1)(x-2)^2$ (B)

16. $\underline{a}.\underline{b} = \frac{2}{3}$ $\underline{a}$ & $\underline{b}$ are unit vectors ie. $|\underline{a}| = |\underline{b}| = 1$

$\underline{a}.(\underline{a} + 2\underline{b})$

$= \underline{a}.\underline{a} + 2\underline{a}.\underline{b}$

$= a^2 + 2.\frac{2}{3}$

$= 1 + \frac{4}{3}$

$= \frac{7}{3}$ (C)

17. $3x^2 + 12x + 17$

$= 3(x^2 + 4x) + 17$

$= 3[(x+2)^2 - 4] + 17$

$= 3(x+2)^2 - 12 + 17$

$= 3(x+2)^2 + 5$

(B)

18. $1 - 2\sin^2 15°$

$= \cos 2(15)°$ using $\cos 2A = 1 - 2\sin^2 A$

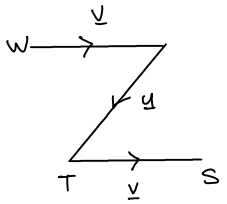
$= \cos 30°$

$= \frac{\sqrt{3}}{2}$

(C)

19. $\overrightarrow{SW} = -\underline{v} - \underline{u} - \underline{v}$

$= -\underline{u} - 2\underline{v}$



(A)

20. $2 - \log_5 \frac{1}{25}$

$= 2 - (-2)$

$= 4$

$\log_5 \frac{1}{25}$ = what power of 5 gives $\frac{1}{25}$?

$\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

$\Rightarrow$ power -2

(D)

21. (a) $y = 3x^2 - x^3$

for stationary points $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 6x - 3x^2$

$6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0, 2$

When $x = 0$, $y = 0$

when $x = 2$, $y = 3(2)^2 - (2)^3$

$y = 12 - 8$

$y = 4$

Stat pts at $(0,0)$ & $(2,4)$

x	$\rightarrow$	0	$\rightarrow$	2	$\rightarrow$
$\frac{dy}{dx}$	$-$	0	$+$	0	$-$
shape	\	—	/	—	\

when $x = -1$: $\frac{dy}{dx} = 6(-1) - 3(-1)^2$, $= -6 - 3$, $m = -9$

when $x = 1$: $\frac{dy}{dx} = 6(1) - 3(1)^2$, $= 6 - 3$, $m = 3$

when $x = 3$: $\frac{dy}{dx} = 6(3) - 3(3)^2$, $= 18 - 27$, $m = -9$

Min TP at $(0,0)$

Max TP at $(2,4)$

(b) on x-axis $y = 0$

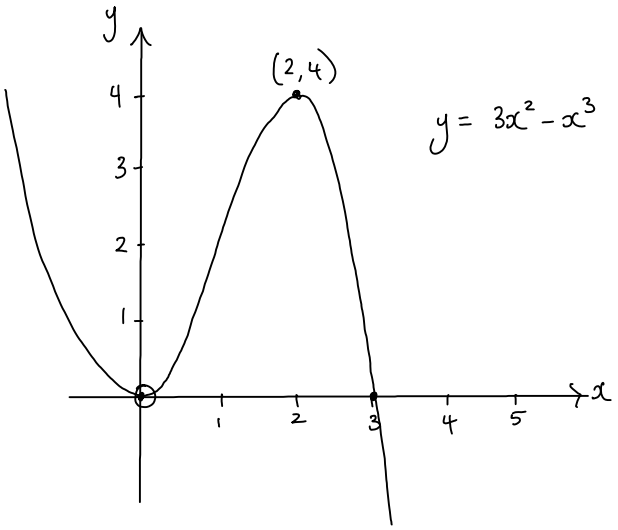
$\Rightarrow 3x^2 - x^3 = 0$

$x^2(3 - x) = 0$

Roots at $x = 0, 3$

on y-axis, $x = 0$

$\Rightarrow y = 0$



22. $6x^3 + 7x^2 + ax + b$

$(x+1)$ is a factor

$\Rightarrow x = -1$ is a root

$x = -1$

	6	7	a	b
		-6	-1	1-a
	6	1	a-1	0

$\Rightarrow b + 1 - a = 0$

$b = a - 1$

72 is the remainder when divided by $(x-2)$

$x = 2$

	6	7	a	b
		12	38	2a+76
	6	19	a+38	72

$\Rightarrow b + 2a + 76 = 72$

$b = -2a - 4$

solve:

$a - 1 = -2a - 4$

$3a = -3$

$a = -1$

$b = (-1) - 1$

$b = -2$

$a = -1$ and $b = -2$

(b) Using first quotient:

$6x^2 + x + (a-1) = 6x^2 + x - 2$

$\Rightarrow (x+1)(6x^2 + x - 2)$

$= (x+1)(3x+2)(2x-1)$

23. (a) $x^2 + y^2 + 2x - 4y - 15 = 0$ $\quad y = 3x - 5$

for points of intersection $y = y$

$x^2 + (3x-5)^2 + 2x - 4(3x-5) - 15 = 0$

$x^2 + 9x^2 - 30x + 25 + 2x - 12x + 20 - 15 = 0$

$10x^2 - 40x + 30 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1, 3$

$\Rightarrow P(1,-2)$ and $Q(3,4)$

when $x = 1$

$y = 3(1) - 5$

$y = -2$ $\quad (1,-2)$

when $x = 3$

$y = 3(3) - 5$

$y = 4$ $\quad (3,4)$

(b) $T(-1,2)$ $P(1,-2)$ $Q(3,4)$

$$m_{PT} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{QT} = \frac{y_2 - y_1}{x_2 - x_1}$$

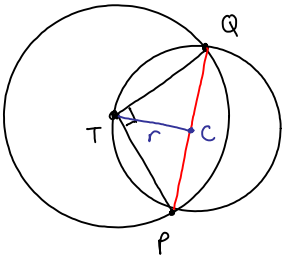
$$= \frac{-2-2}{1-(-1)} \qquad = \frac{4-2}{3-(-1)}$$

$$= \frac{-4}{2} \qquad = \frac{2}{4}$$

$$m_{PT} = -2 \qquad m_{QT} = \frac{1}{2}$$

$m_{PT} \times m_{QT} = -1 \Rightarrow PT \perp QT$

(c)



As $PT \perp QT$ then PQ is diameter of C_2.

Centre of C_2 will be midpoint of PQ

$C(2,1)$

$$r^2 = (2-(-1))^2 + (1-2)^2$$

$$r^2 = 9 + 1$$

$$r^2 = 10$$

Equation of C_2 is $(x-2)^2 + (y-1)^2 = 10$

24.

$$y = ka^x$$

$$\log y = \log ka^x$$

$$\log y = \log k + \log a^x$$

$$\log y = x\log a + \log k$$

$$Y = mx + c$$

$\log_9 a = m$ $\qquad$ $\log_9 k = c$

$\log_9 a = \frac{1}{2}$ $\qquad$ $\log_9 k = 2$

$a = 9^{1/2}$ $\qquad$ $k = 9^2$

$a = 3$ $\qquad$ $k = 81$

from graph: $m = \frac{5-2}{6-0}$ $\qquad$ $c = 2$

$m = \frac{3}{6}$

$m = \frac{1}{2}$

$Y = mx + c$

$Y = \frac{1}{2}x + 2$

(where $Y = \log_9 y$)