HIGHER MATHS 2014 PAPER 2 SOLUTIONS

1. (a) $M(4,1)$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-0}{5-3}$$

$$= \frac{2}{2}$$

$$m_{AB} = 1$$

$$\Rightarrow m_{\perp} = -1$$ as $m_1 m_2 = -1$ for $\perp$ lines

$$y - b = m(x - a)$$

$$y - 1 = -(x-4)$$

$$y - 1 = -x + 4$$

$$y = 5 - x$$

(b) for points of intersection $y = y$

$$\Rightarrow (5-x) + 2x = 6$$

$$5 - x + 2x = 6$$

$$x = 1$$

when $x = 1$

$$y = 5 - (1)$$

$$y = 4$$

$T(1,4)$

(c)

$$m_{AT} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-4}{3-1}$$

$$= \frac{-4}{2}$$

$$m_{AT} = -2$$

or from $y = -2x + 6$

$$m_{AT} = -2$$

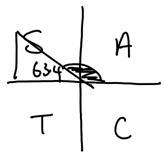
$$\tan\theta = m$$

$$\tan\theta = -2$$

$$\theta = \tan^{-1}(-2)$$

acute angle $= 63.4°$

$$\theta = 116.6°$$



2. $$y = x^4 - 2x^3 + 5$$

$$\frac{dy}{dx} = 4x^3 - 6x^2$$

when $x = 2$

$$y = (2)^4 - 2(2)^3 + 5$$

$$y = 5$$

$(2,5)$

$$\frac{dy}{dx} = 4(2)^3 - 6(2)^2$$

$$= 32 - 24$$

$$m = 8$$

$$y - b = m(x-a)$$

$$y - 5 = 8(x-2)$$

$$y - 5 = 8x - 16$$

$$y = 8x - 11$$

3. (a) $f(x) = x(x-1) + q \qquad g(x) = x + 3$

$$f(g(x)) = f(x+3)$$
$$= (x+3)((x+3)-1) + q$$
$$= (x+3)(x+2) + q$$
$$= x^2 + 5x + 6 + q$$

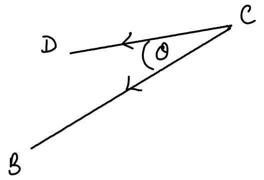
(b) for equal roots $b^2 - 4ac = 0$

$$x^2 + 5x + (6+q) = 0$$
$$a = 1 \quad b = 5 \quad c = 6+q$$
$$(5)^2 - 4(1)(6+q) = 0$$
$$25 - 24 - 4q = 0$$
$$1 - 4q = 0$$
$$1 = 4q$$
$$q = \frac{1}{4}$$

4. (a) $C(11, 12, 6)$ & $D(8, 8, 4)$

(b) $\vec{CB} = \begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix} \qquad \vec{CD} = \begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$

(c)



$$|\vec{CB}| = \sqrt{0^2 + (-8)^2 + (-4)^2}$$
$$= \sqrt{80}$$
$$|\vec{CD}| = \sqrt{(-3)^2 + (-4)^2 + (-2)^2}$$
$$= \sqrt{29}$$

$$\vec{CB} \cdot \vec{CD} = 0.(-3) + (-8)(-4) + (-4)(-2)$$
$$= 40$$

$$\cos\theta = \frac{\vec{CB} \cdot \vec{CD}}{|\vec{CB}||\vec{CD}|}$$
$$\cos\theta = \frac{40}{\sqrt{80}\sqrt{29}}$$
$$\theta = \cos^{-1}\left(\frac{40}{\sqrt{80}\sqrt{29}}\right)$$
$$\theta = 33.9^\circ$$

5. $$\int_4^t (3x+4)^{-\frac{1}{2}}\,dx = 2$$

$$\left[\frac{(3x+4)^{1/2}}{\frac{1}{2}\cdot 3}\right]_4^t = 2$$

$$\left[\frac{2}{3}\sqrt{3x+4}\right]_4^t = 2$$

$$\frac{2}{3}\sqrt{3t+4} - \frac{2}{3}\sqrt{16} = 2$$

$$\frac{2}{3}\sqrt{3t+4} - \frac{8}{3} = 2$$

$$2\sqrt{3t+4} - 8 = 6$$

$$2\sqrt{3t+4} = 14$$

$$\sqrt{3t+4} = 7$$

$$3t + 4 = 49$$

$$3t = 45$$

$$t = 15$$

6. $$\sin x - 2\cos 2x = 1 \qquad 0 \leqslant x < 2\pi$$

$$\sin x - 2(1 - 2\sin^2 x) = 1$$

$$\sin x - 2 + 4\sin^2 x = 1$$

$$4\sin^2 x + \sin x - 3 = 0$$

$$4x^2 + x - 3 = (4x-3)(x+1)$$

$$(4\sin x - 3)(\sin x + 1) = 0$$

either $4\sin x - 3 = 0$ or $\sin x + 1 = 0$

$\sin x = \frac{3}{4}$

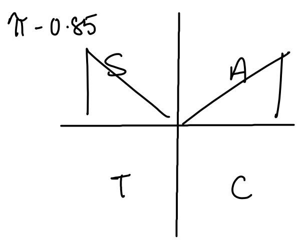
acute angle $= \sin^{-1}\left(\frac{3}{4}\right)$

$= 0.85$ rads

$x = 0.85,\ 2.29$

$\sin x = -1$

$x = \frac{3\pi}{2}$



$x = 0.85,\ 2.29,\ \frac{3\pi}{2}$ (or 4.71) radians

7. (a) for points of intersection $y = y$

$$2x = 6x - x^2$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0, 4$$

$$\int_0^4 (6x - x^2 - 2x)\,dx$$

$$= \int_0^4 (4x - x^2)\,dx$$

$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= \left(2(4)^2 - \frac{(4)^3}{3}\right) - 0$$

$$= 32 - \frac{64}{3}$$

$$= \frac{96}{3} - \frac{64}{3}$$

$$= \frac{32}{3} \text{ units}^2$$

Area of land $= \frac{32}{3} \times 300\,m^2$

$= 3200\,m^2$

(b) $m_T = 2$

$$\frac{dy}{dx} = 6 - 2x$$

find point where $\frac{dy}{dx} = 2$

$\Rightarrow$ $6 - 2x = 2$

$$-2x = -4$$

$$x = 2$$

when $x = 2$,

$y = 6(2) - (2)^2$

$y = 8$ $(2, 8)$

equation of tangent:

$y - b = m(x - a)$

$y - 8 = 2(x - 2)$

$y - 8 = 2x - 4$

$y = 2x + 4$

$$\text{Area} = \int_0^2 \left(2x + 4 - (6x - x^2)\right) dx$$

$$= \int_0^2 (x^2 - 4x + 4)\,dx$$

$$= \left[\frac{x^3}{3} - 2x^2 + 4x\right]_0^2$$

$$= \left(\frac{(2)^3}{3} - 2(2)^2 + 4(2)\right) - 0$$

$$= \frac{8}{3} - 8 + 8$$

$$= \frac{8}{3} \text{ units}^2$$

$\Rightarrow$ Car Park area $= \frac{8}{3} \times 300\,m^2$

$= 800\,m^2$

8. $x^2 + y^2 - 2px - 4py + 3p + 2 = 0$

represents a circle if $g^2 + f^2 - c > 0$

$g = p \quad f = 2p \quad c = 3p + 2$

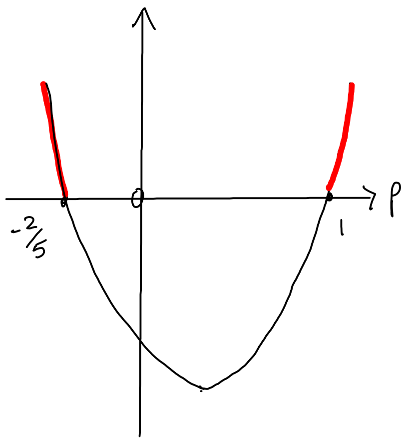
$$p^2 + (2p)^2 - (3p+2) > 0$$
$$p^2 + 4p^2 - 3p - 2 > 0$$
$$5p^2 - 3p - 2 > 0$$

Find roots: $(5p+2)(p-1) = 0$

$5p + 2 = 0$ or $p - 1 = 0$

$p = -\frac{2}{5} \qquad p = 1$

Sketch:



Solution:

$p < -\frac{2}{5}$ or $p > 1$

9. (a) $v(t) = 8\cos\left(2t - \frac{\pi}{2}\right)$

$a(t) = v'(t)$

$= -8\sin\left(2t - \frac{\pi}{2}\right).2$

$= -16\sin\left(2t - \frac{\pi}{2}\right)$

(b) $a(10) = -16\sin\left(2(10) - \frac{\pi}{2}\right)$

$= 6{\cdot}5$

$a(10) > 0 \Rightarrow$ velocity is increasing

(c) $s(t) = \int v(t)\,dt$

$= \int 8\cos\left(2t - \frac{\pi}{2}\right)dt$

$= \dfrac{8\sin\left(2t - \frac{\pi}{2}\right)}{2} + c$

$s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + c$

$s(0) = 4$

$\Rightarrow \quad 4\sin\left(2(0) - \frac{\pi}{2}\right) + c = 4$

$4\sin\left(-\frac{\pi}{2}\right) + c = 4$

$4(-1) + c = 4$

$c - 4 = 4$

$c = 8$

$\Rightarrow \quad s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$