

1 (a) Calculate the values of u_1, u_2, \dots, u_5 for these recurrence relations:

(i) $u_0 = 4 \quad u_{n+1} = 2u_n - 3$

(ii) $u_0 = 64 \quad u_{n+1} = \frac{1}{2}u_n$

(iii) $u_0 = 81 \quad u_{n+1} = -\frac{1}{3}u_n$

(b) For each of the above, explain what happens to u_n as $n \rightarrow \infty$.

2 A retired person has £150 000 invested in a pension fund. This fund earns interest at 5% per annum applied at the end of the year. The canny pensioner withdraws £15 000 for living expenses for the following year immediately after the interest has been added.

Calculate the value of the fund at the beginning of each year.

How many years will the fund last?

3 Find a suitable recurrence relation for these sequences:

(a) 2, 5, 8, 11, 14, ...

(b) 4, 9, 19, 39, 79, ...

4 Determine algebraically the limit for these recurrence relations:

(a) $u_0 = 40, \quad u_{n+1} = 0.8u_n + 20$

(b) $u_0 = 4, \quad u_{n+1} = 60 - 0.5u_n$

5 At 12noon, a hospital patient is given a pill containing 50 units of antibiotic. By 1pm the number of units in the patient's body has dropped by 12%. By 2pm a further 12% of the units remaining at 1pm is lost.

If this fall-off rate is maintained, find the number of units of antibiotic remaining at 6pm.

A doctor considers prescribing a course of treatment which involves a patient taking one of these pills every 6 hours over a long period of time.

The doctor knows that more than 100 units of this antibiotic in the body are regarded as too dangerous.

Should the doctor prescribe this treatment? **Give reasons for your answer.**