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**Mathematics**  
**Higher Prelim Examination 2012/2013**  
**Paper 1**  
**Assessing Units 1 & 2**

**NATIONAL  
QUALIFICATIONS**

**Time allowed - 1 hour 30 minutes**

**Read carefully**

**Calculators may NOT be used in this paper.**

**Section A - Questions 1 - 20 (40 marks)**

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

**Section B (30 marks)**

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

## Read carefully

- 1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2012/2013 (Section A)**.
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- 3 Make sure you write your **name, class and teacher** on the answer sheet provided.
- 4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
- 5 There is **only one correct** answer to each question.
- 6 Rough working should **not** be done on your answer sheet.
- 7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

## Sample Question

A line has equation  $y = 4x - 1$ .

If the point  $(k, 7)$  lies on this line, the value of  $k$  is

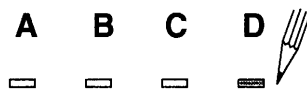
- A 2
- B 27
- C 1.5
- D -2

The correct answer is **A**  $\rightarrow$  2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



## Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

**SECTION A**

**ALL questions should be attempted**

1. If  $f(x) = \frac{2}{x^2}$ , where  $x \neq 0$ , then  $f'(2)$  equals
- A  $-1$   
B  $\frac{1}{2}$   
C  $-32$   
D  $-\frac{1}{2}$
2. When  $x^3 - x^2 + 1$  is divided by  $(x - 3)$  the remainder is
- A  $-35$   
B  $7$   
C  $19$   
D  $13$
3. P and Q have coordinates  $(1, 2)$  and  $(3, k)$  respectively.  
Both points lie on the curve with equation  $y = 2x^2$ .  
The gradient of the line PQ is
- A  $\frac{1}{8}$   
B  $8$   
C  $18$   
D  $12$
4. The exact value of  $\tan \frac{5\pi}{6}$  is
- A  $-\frac{1}{\sqrt{3}}$   
B  $\sqrt{3}$   
C  $1$   
D  $\frac{1}{\sqrt{3}}$

5. A straight line passes through the point  $(0, -4)$  and is parallel to the line with equation  $2x + 6y = 9$ .

Its equation is

A  $y = -\frac{1}{3}x - 4$

B  $y = -3x - 4$

C  $y = \frac{1}{3}x - 4$

D  $2x + 6y = -4$

6. The limit of the sequence defined by the recurrence relation  $U_{n+1} = pU_n + (1 - p^2)$  where  $p$  is a constant and  $0 < p < 1$  is

A  $p$

B  $1 - p$

C  $1 + p$

D  $\frac{1}{1 + p}$

7. A circle has as its equation  $(x - 9)^2 + (y - 2)^2 = 9$ .

Which of the following statements is true?

A the circle intersects the  $x$ -axis at two distinct points.

B the circle does not intersect or touch either of the two axes.

C the circle touches the  $y$ -axis at a single point.

D the circle intersects the  $y$ -axis at two distinct points.

8. A curve for which  $\frac{dy}{dx} = 6x$  passes through the point  $(1, 5)$ .

The particular solution is

A  $y = 6x + 5$

B  $y = 3x^2 + 2$

C  $y = 3x^2 - 1$

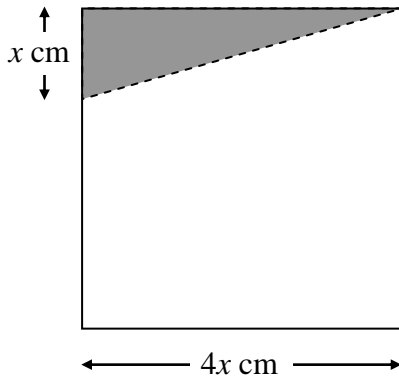
D  $y = 6$

9. The point  $T(0, 6)$  lies on the circumference of the circle with centre  $C$  and equation  $x^2 + y^2 + 2x - 6y + 21 = 0$ .

The gradient of the radius  $CT$  is

- A  $-3$
- B  $\frac{1}{3}$
- C  $3$
- D  $-9$

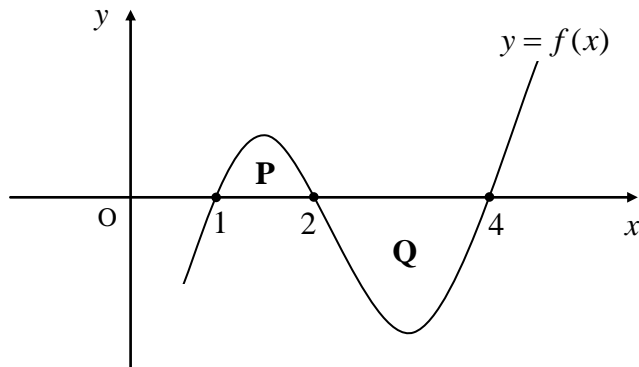
10. The diagram below shows a square of side  $4x$  centimetres.



The ratio of the shaded area to the unshaded area is

- A  $2 : 7$
  - B  $1 : 7$
  - C  $1 : 8$
  - D  $1 : 4$
11. How many real roots does the equation  $(x^2 - 1)(x^2 + 5x + 7) = 0$  have?
- A  $1$
  - B  $2$
  - C  $3$
  - D  $4$

12. Part of the curve  $y = f(x)$  is shown below.



Area **P** is  $\frac{5}{6}$  square units and area **Q** is  $1\frac{5}{6}$  square units.

Which of the following is/are true?

- (1)  $\int_1^2 f(x) dx = \frac{5}{6}$
- (2)  $\int_2^4 f(x) dx = 1\frac{5}{6}$
- (3)  $\int_1^4 f(x) dx = 2\frac{2}{3}$

- A** (1) only
- B** (2) only
- C** (1) and (2) only
- D** (1), (2) and (3)

13. The gradient of the tangent to the curve  $y = (x + 3)^2$  at the point where  $x = 2$  is

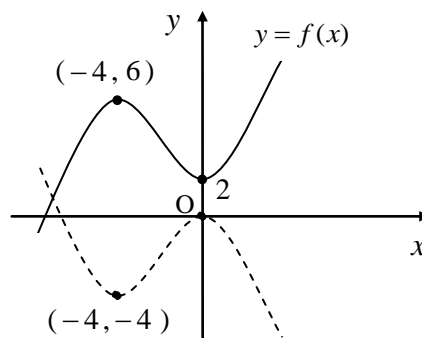
- A** 25
- B** 19
- C**  $32\frac{2}{3}$
- D** 10

14. If  $n = p(p+1)(p+2)$  where  $p$  is a whole number, which of the following must be true?

- (1)  $n$  must be even
- (2)  $n$  is a multiple of 3
- (3)  $n$  is a multiple of 4

- A (1) only
- B (2) only
- C (1) and (2) only
- D (1) and (3) only

15. The diagram shows part of the graph of  $y = f(x)$  as a full line and a part graph of a related function as a broken line.



The equation of the related function is

- A  $y = 2 - f(x)$
- B  $y = f(-x) + 2$
- C  $y = -f(x) - 2$
- D  $y = -f(x + 2)$



16. Given that  $(x+1)^{\frac{1}{3}} = 3$ , then  $x-1$  equals

- A 7
- B 26
- C 25
- D 27

17. Given that  $\cos 2x = k$ , then  $\sin^2 x$  equals

- A  $\frac{1}{2}(k-1)$
- B  $\frac{1}{2}(1+k)$
- C  $1-k^2$
- D  $\frac{1}{2}(1-k)$

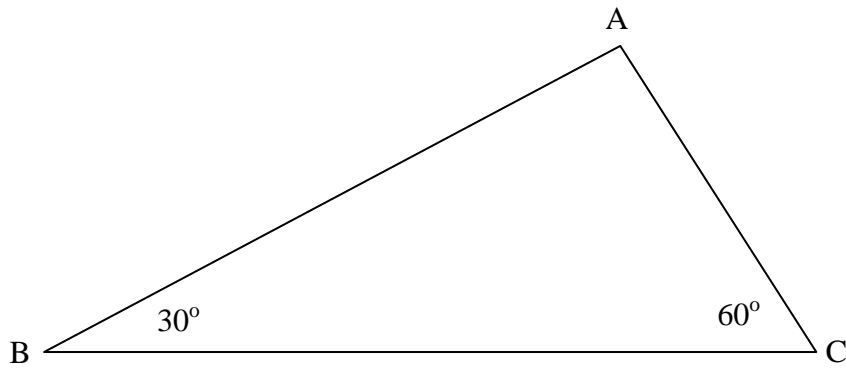
18. If the minimum value of  $x^2 + 4x + a$  is 10, where  $x \in R$ , then  $a$  is

- A 6
- B -4
- C 14
- D 10

19. Given that  $k$  is a constant of integration  $\int \frac{6}{x^4} dx$  equals

- A  $\frac{-2}{x^3} + k$
- B  $\frac{-6}{5x^5} + k$
- C  $\frac{6}{x^3} + k$
- D  $\frac{6x^5}{5} + k$

20. Consider the triangle below.



The ratio of AB : AC is

- A  $\sqrt{2} : 1$
- B  $2 : 1$
- C  $1 : \sqrt{3}$
- D  $\sqrt{3} : 1$

[ END OF SECTION A ]

## SECTION B

**ALL questions should be attempted**

**21.** Two functions are defined on suitable domains as

$$f(x) = 2x - 1 \text{ and } h(x) = \frac{2(x^2 + 1)}{k}.$$

(a) Find  $f(f(x))$  in its simplest form. **2**

(b) Hence show clearly that the equation  $h(x) = f(f(x))$  can be written as

$$2x^2 - 4kx + (3k + 2) = 0 \quad \mathbf{3}$$

(c) Find the two possible values of  $k$  so that  $h(x) = f(f(x))$  has **equal roots**. **5**

**22.** A(1, 2), B(7, 6) and C(-7, 14) are the vertices of a triangle.

(a) Show clearly that triangle ABC is right-angled at A. **3**

(b) M is the mid-point of AC. Find the equation of the median BM. **3**

(c) If the median BM is extended it meets the line through C, with gradient 5, at P.

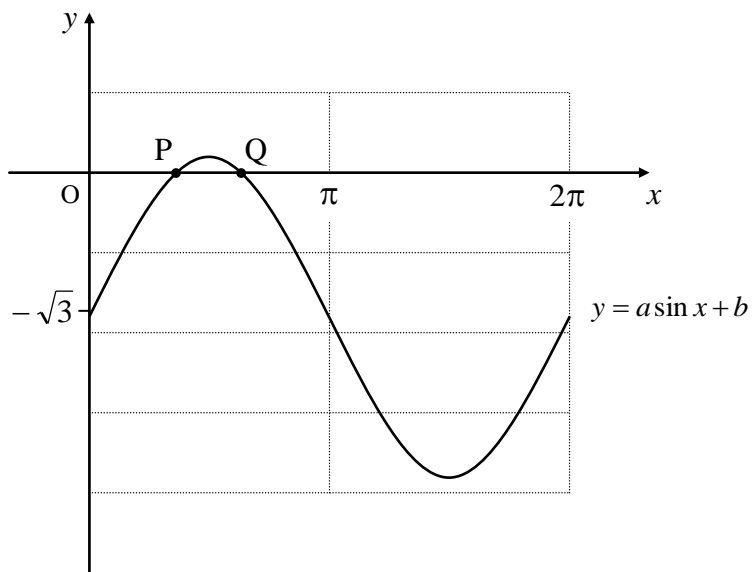
Find the coordinates of P. **4**

**23.** A function is given as  $f(x) = x^3 - 4x + 14$ .

(a) If  $f(p) = 14$ , find the value of  $p$  if  $p > 0$ . **2**

(b) Prove that the function is increasing when  $p$  takes this value. **3**

24. The diagram shows the graph of  $y = a \sin x + b$  for  $0 \leq x \leq 2\pi$ .



- (a) Write down the values of  $a$  and  $b$  (where  $a$  is a whole number). 2
- (b) Determine the exact  $x$ -coordinates for the points P and Q.

**Your answers must be accompanied by the appropriate working.** 3

[ END OF SECTION B ]

[ END OF QUESTION PAPER 1 ]

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**Mathematics**  
**Higher Prelim Examination 2012/2013**  
**Paper 2**  
**Assessing Units 1 & 2**

**NATIONAL  
QUALIFICATIONS**

**Time allowed - 1 hour 10 minutes**

**Read carefully**

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2. Full credit will be given only where the solution contains appropriate working.
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## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

**ALL questions should be attempted**

1. The graph of the curve with equation  $y = 2x^3 - 3x^2 - 3x + a$  crosses the  $x$ -axis at the point  $(-1, 0)$ .

(a) Find the value of  $a$ . **2**

(b) Hence write down the coordinates of the point at which this curve crosses the  $y$ -axis. **1**

(c) This curve also crosses the  $x$ -axis at a further two points.

**Find algebraically** the coordinates of these other two points. **4**

2. Solve **algebraically** the following system of equations for  $0 < x < 90^\circ$ .

$$y = 3 \cos 2x^\circ$$

$$y = 1 - 10 \cos x^\circ$$

**5**

3. A recurrence relation is defined as  $U_{n+1} = 0.8U_n + 14$ .

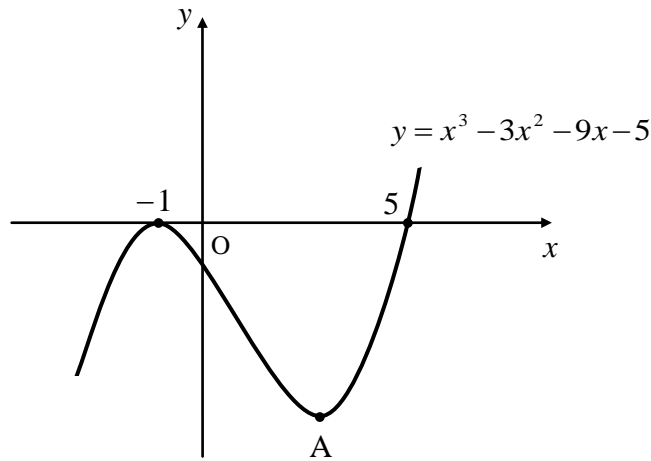
(a) Given that  $U_0 = 20$ , find  $U_3$ . **2**

(b) Express the difference between the limit of this sequence and  $U_3$  as a percentage of the limit. **3**

4. A function is defined on a suitable domain as  $g(x) = \frac{6x^3 - 4}{x^2}$ .

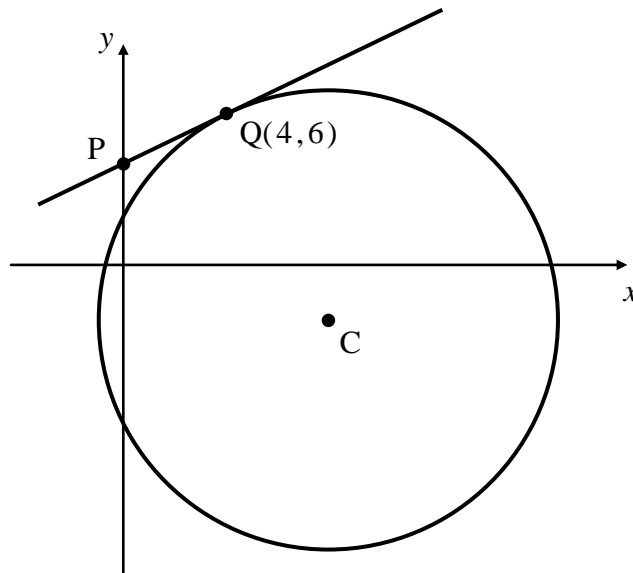
If  $\int_1^a (2x + 4) dx = g'(2)$ , find the value of  $a$  where  $a > 1$ . **7**

5. Part of the graph of  $y = x^3 - 3x^2 - 9x - 5$  is shown in the diagram below.  
 The curve crosses the  $x$ -axis at the points  $(-1, 0)$  and  $(5, 0)$  as shown.



- (a) Find the coordinates of the stationary point  $A$ . 4
- (b) Calculate the finite area enclosed between this curve and the  $x$ -axis. 5

6. The diagram below shows a circle, centre  $C$ , with equation  $x^2 + y^2 - 16x + 4y - 12 = 0$ .  
 The point  $Q(4, 6)$  lies on the circumference of the circle.  
 The line  $PQ$  is a tangent to the circle.

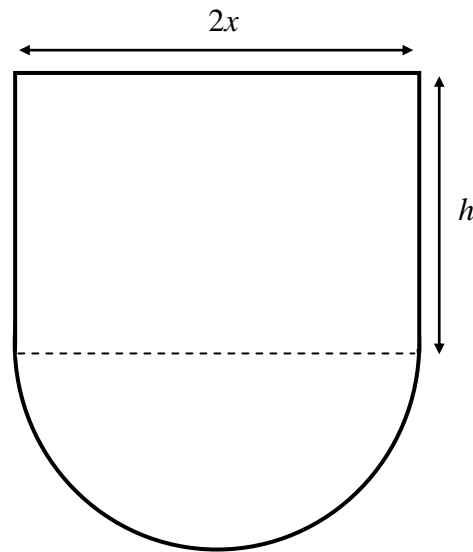


- (a) Find the equation of the tangent  $PQ$ . 4
- (b) Write down the coordinates of  $P$ . 2
- (c) Establish the equation of the circle which passes through the points  $P$ ,  $Q$  and  $C$ . 5



7. An underground storm drain has a cross-section in the shape of a rectangle with a semi-circular base.

The rectangle part of the drain measures  $2x$  metres by  $h$  metres as shown.



- (a) Show clearly that if the **perimeter** of the drain is approximately 3.8 metres then  $h$  can be expressed as

$$h = 1.9 - x - \frac{1}{2}\pi x. \quad 2$$

- (b) Hence show that the cross-sectional area,  $A$ , in terms of  $x$ , can be written as

$$A(x) = 3.8x - 2x^2 - \frac{1}{2}\pi x^2. \quad 2$$

- (c) Find the value of  $x$  which will produce the **largest** cross-sectional area. Justify your answer. 5

8. Angle  $A$  is such that  $\tan A = \frac{1}{\sqrt{5}}$  and both  $A$  and  $2A$  are acute.

- (a) Show clearly that the exact value of  $\sin 2A$  is  $\frac{\sqrt{5}}{3}$ . 3

- (b) Given now that  $\cos 2A = \frac{2}{3}$ , show clearly that  $\sin(\frac{\pi}{3} + 2A)$  has an exact value of  $\frac{1}{6}(2\sqrt{3} + \sqrt{5})$ . 4

[ END OF QUESTION PAPER ]

**Paper 1 - Section A - Answer Sheet**

NAME :

CLASS :

TEACHER :

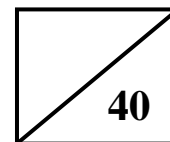
You should use an HB pencil.  
 Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

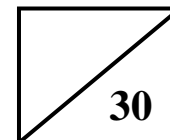
A	B	C	D
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
13	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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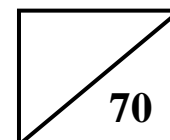
Section A



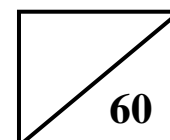
Section B



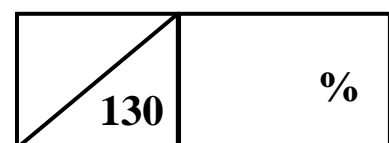
Total (P1)



Total (P2)



Overall Total



*Please make sure you have filled in all your details above before handing in this answer sheet.*

- 1 D
- 2 C
- 3 B
- 4 A
- 5 A
- 6 C
- 7 A
- 8 B
- 9 C
- 10 B
- 11 B
- 12 A
- 13 D
- 14 C
- 15 A
- 16 C
- 17 D
- 18 C
- 19 A
- 20 D

	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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6	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
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11	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
13	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
14	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
15	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
16	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
17	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
18	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
19	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
20	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	ans: $4x - 3$ (2 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes</li> <li>•<sup>2</sup> simplifies</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(f(x)) = 2(2x - 1) - 1</math></li> <li>•<sup>2</sup> <math>f(f(x)) = 4x - 3</math></li> </ul>
(b)	ans: proof (3 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> equates expressions</li> <li>•<sup>2</sup> cross multiplies</li> <li>•<sup>3</sup> multiplies and rearranges to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{2(x^2 + 1)}{k} = 4x - 3</math></li> <li>•<sup>2</sup> <math>2(x^2 + 1) = k(4x - 3);</math></li> <li>•<sup>3</sup> <math>2x^2 + 2 - 4kx + 3k</math></li> </ul>
(c)	ans: $k = -\frac{1}{2}$ or 2 (5 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> knows condition for equal roots</li> <li>•<sup>2</sup> substitutes into <math>b^2 - 4ac</math></li> <li>•<sup>3</sup> simplifies</li> <li>•<sup>4</sup> factorises</li> <li>•<sup>5</sup> solves for <math>k</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>b^2 - 4ac = 0</math> [stated or implied]</li> <li>•<sup>2</sup> <math>(-4k)^2 - 4.2.(3k + 2) = 0</math></li> <li>•<sup>3</sup> <math>16k^2 - 24k - 16 = 0;</math></li> <li>•<sup>4</sup> <math>8(2k + 1)(k - 2) = 0</math></li> <li>•<sup>5</sup> <math>k = -\frac{1}{2}</math> or 2</li> </ul>
22(a)	ans: proof (3 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to find gradients of AB and AC</li> <li>•<sup>2</sup> finds gradients of AB and AC</li> <li>•<sup>3</sup> communicates conclusion</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> evidence of finding gradients of two lines</li> <li>•<sup>2</sup> <math>m_{AB} = \frac{2}{3}; m_{AC} = -\frac{3}{2}</math></li> <li>•<sup>3</sup> since <math>m_{AB} \times m_{AC} = -1</math> angle A is right</li> </ul>
(b)	ans: $5y + x = 37$ (3 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> finds midpoint of AC</li> <li>•<sup>2</sup> finds gradient of BM</li> <li>•<sup>3</sup> substitute</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> M(-3, 8)</li> <li>•<sup>2</sup> <math>m_{BM} = -\frac{1}{5}</math></li> <li>•<sup>3</sup> <math>y - 8 = -\frac{1}{5}(x + 3)</math> or <math>y - 6 = -\frac{1}{5}(x - 7)</math></li> </ul>
(b)	ans: P(-8, 9) (4 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> finds equation of line through C</li> <li>•<sup>2</sup> knows to use system of equations</li> <li>•<sup>3</sup> finds values for <math>x</math> and <math>y</math></li> <li>•<sup>4</sup> states coordinates of P</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y - 14 = 5(x + 7) \Rightarrow y - 5x = 49</math></li> <li>•<sup>2</sup> evidence of scaling</li> <li>•<sup>3</sup> <math>x = -8; y = 9</math></li> <li>•<sup>4</sup> P(-8, 9)</li> </ul>

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	<b>ans: <math>p = 2</math> (2 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> equates expressions and takes terms to LHS</li> <li>•<sup>2</sup> factorises, solves and discards</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>p^3 - 4p + 14 = 14 \Rightarrow p^3 - 4p = 0</math></li> <li>•<sup>2</sup> <math>p(p^2 - 4) = 0 \Rightarrow p = 2</math></li> </ul>
(b)	<b>ans: proof (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to take derivative</li> <li>•<sup>2</sup> substitutes <math>p = 2</math></li> <li>•<sup>3</sup> communicates</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f'(p) = 3p^2 - 4</math></li> <li>•<sup>2</sup> <math>f'(2) = 3(2)^2 - 4 = 8</math></li> <li>•<sup>3</sup> derivative <math>&gt; 0</math> so function is increasing</li> </ul>
24(a)	<b>ans: <math>a = 2; b = -\sqrt{3}</math> (2 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> states value of <math>a</math></li> <li>•<sup>2</sup> states value of <math>b</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>a = 2</math></li> <li>•<sup>2</sup> <math>b = -\sqrt{3}</math></li> </ul>
(b)	<b>ans: <math>\pi/3; 2\pi/3</math> (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> equates equation to zero</li> <li>•<sup>2</sup> finds <math>x</math> – coordinate of P</li> <li>•<sup>3</sup> finds <math>x</math> – coordinate of Q</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2\sin x - \sqrt{3} = 0</math></li> <li>•<sup>2</sup> <math>\frac{\pi}{3}</math></li> <li>•<sup>3</sup> <math>\frac{2\pi}{3}</math></li> </ul>

Total: 70 marks
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	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	<p>ans: <math>a = 2</math> (2 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes into equation</li> <li>•<sup>2</sup> solves for <math>a</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2(-1)^3 - 3(-1)^2 - 3(-1) + a = 0</math></li> <li>•<sup>2</sup> <math>a = 2</math></li> </ul>
(b)	<p>ans: <math>(0, 2)</math> (1 mark)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> states coordinates of <math>y</math> - intercept</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(0, 2)</math></li> </ul>
(c)	<p>ans: <math>(2, 0); (\frac{1}{2}, 0)</math> (4 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> equates equation to zero</li> <li>•<sup>2</sup> knows to use synthetic division</li> <li>•<sup>3</sup> factorises and solves for <math>x</math></li> <li>•<sup>4</sup> states coordinates of two points</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x^3 - 3x^2 - 3x + 2 = 0</math></li> <li>•<sup>2</sup> <math display="block">\begin{array}{r rrrr} 2 &amp; 2 &amp; -3 &amp; -3 &amp; 2 \\ &amp; &amp; 4 &amp; 2 &amp; -2 \\ \hline &amp; 2 &amp; 1 &amp; -1 &amp; 0 \end{array}</math> </li> <li>•<sup>3</sup> <math>(x - 2)(2x - 1)(x + 1); x = 2, \frac{1}{2}, -1</math></li> <li>•<sup>4</sup> <math>(2, 0); (\frac{1}{2}, 0)</math></li> </ul>
2	<p>ans: <math>70.5^\circ</math> (5 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> equates two equations</li> <li>•<sup>2</sup> replaces <math>\cos 2x^\circ</math></li> <li>•<sup>3</sup> multiplies out and rearranges to trinomial</li> <li>•<sup>4</sup> factorises</li> <li>•<sup>5</sup> solves for other bracket and discards</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>3 \cos 2x^\circ = 1 - 10 \cos x^\circ</math></li> <li>•<sup>2</sup> <math>3(2 \cos^2 x - 1) = 1 - 10 \cos x^\circ</math></li> <li>•<sup>3</sup> <math>6 \cos^2 x + 10 \cos x^\circ - 4 = 0</math></li> <li>•<sup>4</sup> <math>2(3 \cos x^\circ - 1)(\cos x^\circ + 2) = 0</math></li> <li>•<sup>5</sup> <math>70.5^\circ</math></li> </ul>
3(a)	<p>ans: <math>44.4</math> (2 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> strategy of finding consecutive terms</li> <li>•<sup>2</sup> answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds <math>U_1</math> and <math>U_2</math></li> <li>•<sup>2</sup> finds <math>U_3</math></li> </ul>
(b)	<p>ans: <math>36.6\%</math> (3 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> finds limit of sequence</li> <li>•<sup>2</sup> finds difference and percentage</li> <li>•<sup>3</sup> answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>L = \frac{14}{0.2} = 70</math></li> <li>•<sup>2</sup> <math>70 - 44.4 = 25.6; L = \frac{25.6}{70} \times 100\%</math></li> <li>•<sup>3</sup> <math>36.6\%</math></li> </ul>

	Give 1 mark for each •	Illustration(s) for awarding each mark
4	<b>ans: <math>a = 2</math></b> (7 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> finds integral</li> <li>•<sup>2</sup> subs and simplifies</li> <li>•<sup>3</sup> prepares to differentiate</li> <li>•<sup>4</sup> differentiates</li> <li>•<sup>5</sup> evaluates and equates to integral</li> <li>•<sup>6</sup> factorises</li> <li>•<sup>7</sup> discards and solves for <math>a</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>[x^2 + 4x]_1^a</math></li> <li>•<sup>2</sup> <math>a^2 + 4a - 5</math></li> <li>•<sup>3</sup> <math>g(x) = 6x - 4x^{-2}</math></li> <li>•<sup>4</sup> <math>g'(x) = 6 + \frac{8}{x^3}</math></li> <li>•<sup>5</sup> <math>g'(2) = 6 + \frac{8}{8} = 7 \Rightarrow a^2 + 4a - 5 = 7</math></li> <li>•<sup>6</sup> <math>(a + 6)(a - 2) = 0</math></li> <li>•<sup>7</sup> <math>a = 2</math></li> </ul>
5(a)	<b>ans: <math>A(3, -32)</math></b> (4 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to take derivative</li> <li>•<sup>2</sup> differentiates</li> <li>•<sup>3</sup> factorises and solves for <math>x</math></li> <li>•<sup>4</sup> substitutes and states coords of A</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 0</math> for stationary point</li> <li>•<sup>2</sup> <math>3x^2 - 6x - 9 = 0</math></li> <li>•<sup>3</sup> <math>3(x - 3)(x + 1) = 0; x = 3</math></li> <li>•<sup>4</sup> <math>A(3, -32)</math></li> </ul>
(b)	<b>ans: <math>108 \text{ units}^2</math></b> (5 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> uses correct limits</li> <li>•<sup>2</sup> knows how to find area</li> <li>•<sup>3</sup> integrates</li> <li>•<sup>4</sup> substitutes limits</li> <li>•<sup>5</sup> evaluates</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int_{-1}^5 \dots \dots \dots</math></li> <li>•<sup>2</sup> <math>\int_{-1}^5 x^3 - 3x^2 - 9x - 5 \, dx</math></li> <li>•<sup>3</sup> <math>\left[ \frac{x^4}{4} - x^3 - \frac{9x^2}{2} - 5x \right]_{-1}^5</math></li> <li>•<sup>4</sup> <math>\left[ \frac{5^4}{4} - 5^3 - \frac{9(5)^2}{2} - 5(5) \right] - \left[ \frac{(-1)^4}{4} - (-1)^3 - \frac{9(-1)^2}{2} - 5(-1) \right]</math></li> <li>•<sup>5</sup> <math>-108 = 108 \text{ units}^2</math></li> </ul>

	Give 1 mark for each •	Illustration(s) for awarding each mark
6(a)	ans: $2y - x = 8$ (4 marks)	
	<ul style="list-style-type: none"> <li>●<sup>1</sup> finds centre of circle</li> <li>●<sup>2</sup> finds gradient of radius</li> <li>●<sup>3</sup> finds gradient of tangent</li> <li>●<sup>4</sup> substitutes into equation of line</li> </ul>	<ul style="list-style-type: none"> <li>●<sup>1</sup> centre (8, -2)</li> <li>●<sup>2</sup> <math>m_{\text{rad}} = -2</math></li> <li>●<sup>3</sup> <math>m_{\text{tan}} = \frac{1}{2}</math></li> <li>●<sup>4</sup> <math>y - 6 = \frac{1}{2}(x - 4)</math></li> </ul>
	(b) ans: P(0, 4) (2 marks)	
	<ul style="list-style-type: none"> <li>●<sup>1</sup> realises <math>x = 0</math></li> <li>●<sup>2</sup> states coordinates of P</li> </ul>	<ul style="list-style-type: none"> <li>●<sup>1</sup> <math>2y - 0 = 8</math></li> <li>●<sup>2</sup> P(0, 4)</li> </ul>
(c)	ans: $(x + 6)^2 + (y - 1)^2 = 25$ (5 marks)	
	<ul style="list-style-type: none"> <li>●<sup>1</sup> realises angle PQC is right</li> <li>●<sup>2</sup> knows PC is diameter</li> <li>●<sup>3</sup> finds centre of circle</li> <li>●<sup>4</sup> finds radius</li> <li>●<sup>5</sup> subs into equation of a circle</li> </ul>	<ul style="list-style-type: none"> <li>●<sup>1</sup> angle PQC = 90°</li> <li>●<sup>2</sup> PC is diameter</li> <li>●<sup>3</sup> centre is (4, 1)</li> <li>●<sup>4</sup> radius = 5 [(4, 1) → (8, -2) or (0, 4)]</li> <li>●<sup>5</sup> <math>(x - 4)^2 + (y - 1)^2 = 25</math></li> </ul>
7(a)	ans: proof (2 marks)	
	<ul style="list-style-type: none"> <li>●<sup>1</sup> finds expression for perimeter</li> <li>●<sup>2</sup> rearranges to given answer</li> </ul>	<ul style="list-style-type: none"> <li>●<sup>1</sup> <math>2x + 2h + \pi x = 3 \cdot 8</math></li> <li>●<sup>2</sup> <math>2h = 3 \cdot 8 - 2x - \pi x; h = 1 \cdot 9 - x - \frac{1}{2} \pi x</math></li> </ul>
	(b) ans: proof (2 marks)	
	<ul style="list-style-type: none"> <li>●<sup>1</sup> finds expression for area</li> <li>●<sup>2</sup> subs expression for <math>h</math> and rearranges</li> </ul>	<ul style="list-style-type: none"> <li>●<sup>1</sup> <math>A(x) = 2xh + \frac{1}{2} \pi x^2</math></li> <li>●<sup>2</sup> <math>A(x) = 2x(1 \cdot 9 - x - \frac{1}{2} \pi x) + \frac{1}{2} \pi x^2</math></li> <li><math>A(x) = 3 \cdot 8x - 2x^2 - \pi x^2 + \frac{1}{2} \pi x^2</math></li> </ul>
(c)	ans: 0.532m (5 marks)	
	<ul style="list-style-type: none"> <li>●<sup>1</sup> knows to make derivative = 0</li> <li>●<sup>2</sup> differentiates first 2 terms</li> <li>●<sup>3</sup> differentiates last term</li> <li>●<sup>4</sup> solves for <math>x</math></li> <li>●<sup>5</sup> justifies answer</li> </ul>	<ul style="list-style-type: none"> <li>●<sup>1</sup> <math>A'(x) = 0</math></li> <li>●<sup>2</sup> <math>3 \cdot 8 - 4x \dots</math></li> <li>●<sup>3</sup> <math>\dots - \pi x</math></li> <li>●<sup>4</sup> <math>\pi x + 4x = 3 \cdot 8; x = 0.532</math></li> <li>●<sup>5</sup> table of values shown</li> </ul>



	Give 1 mark for each •	Illustration(s) for awarding each mark
8(a)	<b>ans: proof (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> uses expansion for <math>\sin 2A</math></li> <li>•<sup>2</sup> finds values of <math>\sin A</math> and <math>\cos A</math></li> <li>•<sup>3</sup> subs and rearranges to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\sin 2A = 2 \sin A \cos A</math></li> <li>•<sup>2</sup> <math>\sin A = \frac{1}{\sqrt{6}}</math> and <math>\cos A = \frac{\sqrt{5}}{\sqrt{6}}</math></li> <li>•<sup>3</sup> <math>\sin 2A = 2 \times \frac{1}{\sqrt{6}} \times \frac{\sqrt{5}}{\sqrt{6}} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}</math></li> </ul>
(b)	<b>ans: proof (4 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> expands</li> <li>•<sup>2</sup> subs exact values</li> <li>•<sup>3</sup> subs other values</li> <li>•<sup>4</sup> rearranges to given answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\sin\left(\frac{\pi}{3} + 2A\right) = \sin\frac{\pi}{3} \cos 2A + \cos\frac{\pi}{3} \sin 2A</math></li> <li>•<sup>2</sup> <math>\frac{\sqrt{3}}{2} \cos 2A + \frac{1}{2} \sin 2A</math></li> <li>•<sup>3</sup> <math>\frac{\sqrt{3}}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{\sqrt{5}}{3}</math></li> <li>•<sup>4</sup> <math>\frac{2\sqrt{3}}{6} + \frac{\sqrt{5}}{6} = \frac{2\sqrt{3} + \sqrt{5}}{6} = \frac{1}{6}(2\sqrt{3} + \sqrt{5})</math></li> </ul>
		<div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 0 auto;"> <b>Total: 60 marks</b> </div>