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**Mathematics**  
**Higher Prelim Examination 2013/2014**  
**Paper 1**  
**Assessing Units 1 & 2**

**NATIONAL  
QUALIFICATIONS**

**Time allowed - 1 hour 30 minutes**

**Read carefully**

**Calculators may NOT be used in this paper.**

**Section A - Questions 1 - 20 (40 marks)**

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

**Section B (30 marks)**

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

## Read carefully

- 1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2013/2014 (Section A)**.
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- 3 Make sure you write your **name, class and teacher** on the answer sheet provided.
- 4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
- 5 There is **only one correct** answer to each question.
- 6 Rough working should **not** be done on your answer sheet.
- 7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

## Sample Question

A line has equation  $y = 4x - 1$ .

If the point  $(k, 7)$  lies on this line, the value of  $k$  is

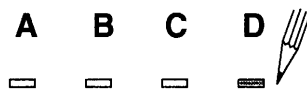
- A 2
- B 27
- C 1.5
- D -2

The correct answer is **A**  $\rightarrow$  2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



## Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

**SECTION A**

**ALL questions should be attempted**

1. The line  $ax + 3y + 1 = 0$  is parallel to the line with equation  $2x + y - 3 = 0$ .  
The value of  $a$  is

- A  $-\frac{1}{3}$
- B 6
- C  $-2$
- D 2

2. The function  $f$  is defined as  $f(x) = \frac{10 - x^2}{x}$ ,  $x \neq 0$ . The value of  $f(f(-2))$  equals

- A  $-\frac{39}{7}$
- B  $\frac{1}{3}$
- C  $-3$
- D  $-\frac{1}{3}$

3. If  $f(x) = \frac{1}{x}$ , where  $x \neq 0$ , then  $f'(2)$  equals

- A 1
- B  $\frac{1}{2}$
- C  $-\frac{1}{4}$
- D  $\frac{1}{4}$

4. A sequence is defined by the recurrence relation  $U_{n+1} = kU_n - 2$  with  $U_0 = 12$ .

An expression in terms of  $k$  for  $U_2$  is

- A  $12k - 2$
- B  $12k^2 - 2k$
- C  $12k^2 - 2k - 2$
- D  $6k^2 - k - 1$

5.  $\cos(\frac{2}{3}\pi - A)$  is equal to

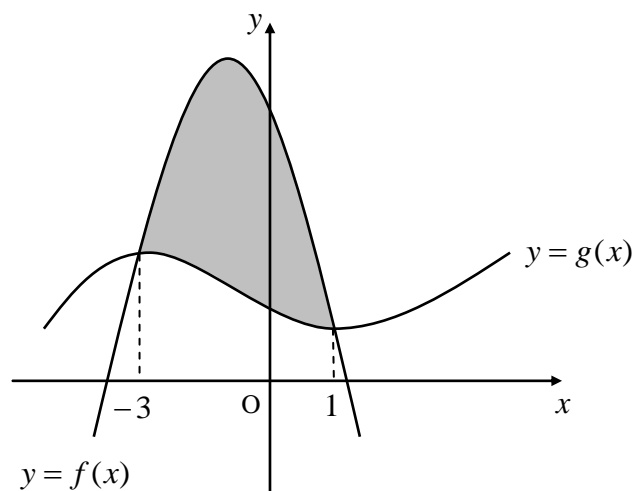
A  $-\frac{1}{2}(\cos A - \sqrt{3} \sin A)$

B  $\frac{1}{2}(\cos A + \sqrt{3} \sin A)$

C  $\frac{1}{2}(\sqrt{3} \cos A + \sin A)$

D  $-\frac{1}{2}(\sqrt{3} \cos A - \sin A)$

6. The diagram shows the area bounded by the curves  $y = f(x)$  and  $y = g(x)$



Which of the following gives the value of the shaded area?

A  $\int_{-3}^1 g(x) + f(x) dx$

B  $\int_1^{-3} f(x) - g(x) dx$

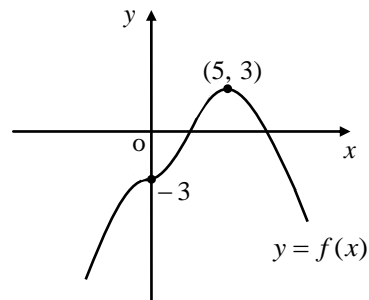
C  $\int_{-3}^1 g(x) - f(x) dx$

D  $\int_{-3}^1 f(x) - g(x) dx$

7. If  $\frac{dy}{dx} > 0$  for **all** values of  $x$  then

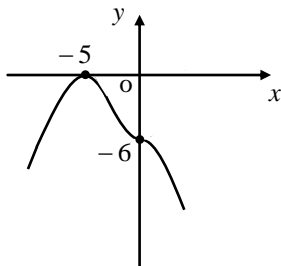
- A  $y > 0$  for all  $x$
- B  $y$  cannot have a maximum value
- C  $y$  is always decreasing
- D  $y < 0$  for all  $x$

8. Part of the graph of the function  $y = f(x)$  is shown opposite.

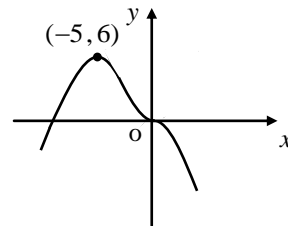


Which of the following graphs could represent the related function  $y = -f(x) - 3$  ?

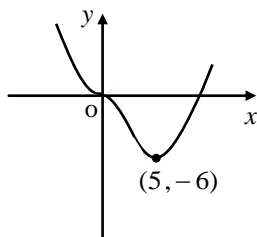
A



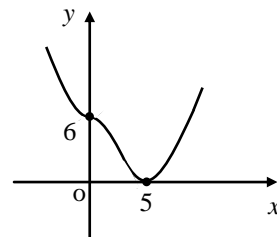
B



C



D



9. Which of the following statements is **false** for the circle  $x^2 + y^2 - 10y = 0$  ?

- A The circle passes through the origin
- B The circle has radius 5 units
- C The circle passes through the point (10,10)
- D The circle has its centre on the y-axis

10.  $\int x^{-\frac{1}{2}} dx$  equals

- A  $x^{\frac{1}{2}} + C$
- B  $2x^{\frac{1}{2}} + C$
- C  $\frac{1}{2}x^{\frac{1}{2}} + C$
- D  $-\frac{1}{2}x^{-\frac{3}{2}} + C$

11. The quadratic equation  $2x^2 - 3x + (k + 1) = 0$  has **no real roots**.

Which one of the following describes the possible values of  $k$ ?

- A  $k < \frac{1}{8}$
- B  $k > \frac{1}{8}$
- C  $k > -\frac{1}{8}$
- D  $k < -\frac{1}{8}$

12. When the polynomial  $3x^3 + 7x^2 - 15$  is divided by  $x + 3$  the remainder is

- A  $-9$
- B  $129$
- C  $33$
- D  $-33$

13. P is the point  $(-4, 0)$ , Q is  $(0, 3)$  and R is  $(-1, 1)$ .

The fourth vertex S of the parallelogram PQRS has coordinates

- A  $(-3, 2)$
- B  $(-5, -2)$
- C  $(5, 2)$
- D  $(3, -2)$

14. If  $\cos x^\circ = -\sin 210^\circ$  for  $0 < x < 180$ , then  $x$  has the value

- A 120
- B 150
- C 30
- D 60

15. An arrow is fired into the air reaching a height of ' $h$ ' metres after ' $t$ ' seconds where  $h(t) = 2 + 18t - 3t^2$ . The time taken, in seconds, to reach its maximum height is

- A 29
- B 3
- C 2
- D 9



16. The line  $y = mx$  passes through the point of intersection of the lines  $x = a$  and  $y = b$ , if and only if

- A  $m = \frac{a}{b}$
- B  $m = -\frac{a}{b}$
- C  $m = \frac{b}{a}$
- D  $m = -\frac{b}{a}$

17. The solution to the inequality  $(3 - x)(2 + x) > 0$  is

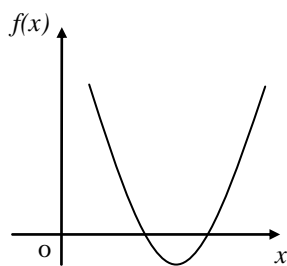
- A  $-3 < x < 2$
- B  $x < -3$  and  $x > 2$
- C  $x < -2$  and  $x > 3$
- D  $-2 < x < 3$



18. If  $\frac{dy}{dx} = 2x - 1$  and  $y = 4$  when  $x = -1$ , then  $y$  is equal to

- A  $x^2 - x$
- B  $x^2 - x + 2$
- C  $x^2 - x + 4$
- D  $x^2 - x + 6$

19. The diagram below shows part of a quadratic function  $f(x) = ax^2 + bx + c$ .



Which one of the following statements is true

- A  $a < 0$
- B  $c < 0$
- C  $b^2 - 4ac = 0$
- D  $b^2 - 4ac > 0$

20. If  $x^{\frac{1}{2}} = 8^{\frac{1}{4}} y^{\frac{3}{4}}$ , then  $y$  equals

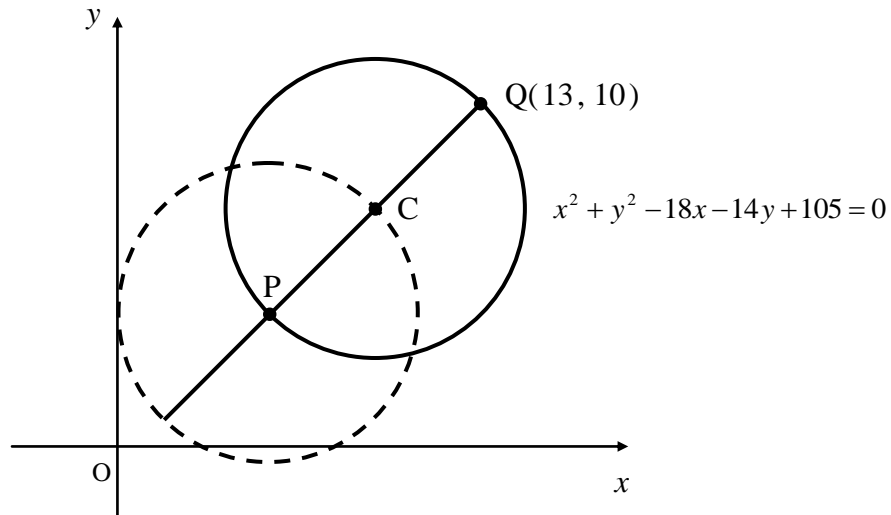
- A  $\frac{x^{\frac{2}{3}}}{2}$
- B  $8^{\frac{1}{2}} y^{\frac{3}{2}}$
- C  $2x^{\frac{2}{3}}$
- D  $\frac{2}{x^{\frac{2}{3}}}$

[ END OF SECTION A ]

**ALL questions should be attempted**

**21.** The diagram below, which is not drawn to scale, shows a circle, centre C, with equation  $x^2 + y^2 - 18x - 14y + 105 = 0$ .

PQ is a diameter of this circle.



- (a) Write down the coordinates of C. **1**
- (b) Hence find the coordinates of P. **2**
- (c) Find the equation of the circle, centre P and passing through C. **3**

**22.** Two functions are defined on suitable domains as  $f(x) = 2x + 1$  and  $g(x) = (x - 2)^2$ .

Given that the function  $h$  is such that  $h(x) = g(f(x))$ , express  $h$  in the form

$$h(x) = a(x + b)^2 + c, \text{ where } a, b \text{ and } c \text{ are integers,}$$

and hence write down the minimum value of  $h$  and the corresponding replacement for  $x$ . **6**

**23.** Solve the equation  $\sqrt{12} \sin 2A - 2\sqrt{6} \cos A = 0$ , for  $0 < A \leq \frac{\pi}{2}$ . **5**

24. A recurrence relation is given as  $u_{n+1} = 0.8u_n + 60$ .
- (a) Given that  $u_1 = 220$ , find the initial value,  $u_0$ , of this sequence. 2
- (b) State why this sequence has a limit and hence find the **difference** between the initial value and the limit of the sequence. 4
- 
25. An equation is given as  $x^3 + ax^2 - x + b = 0$ , where  $a$  and  $b$  are constants.
- (a) It is known that  $x = 1$  and  $x = -2$  are two roots of this equation.  
Use the above roots to establish the values of the constants  $a$  and  $b$ . 4
- (b) Hence find the third root of this equation. 3

[ END OF SECTION B ]

[ END OF QUESTION PAPER 1 ]

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**Mathematics**  
**Higher Prelim Examination 2013/2014**  
**Paper 2**  
**Assessing Units 1 & 2**

**NATIONAL  
QUALIFICATIONS**

**Time allowed - 1 hour 10 minutes**

**Read carefully**

- 1. Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Scalar Product:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}.$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

### Table of standard derivatives:

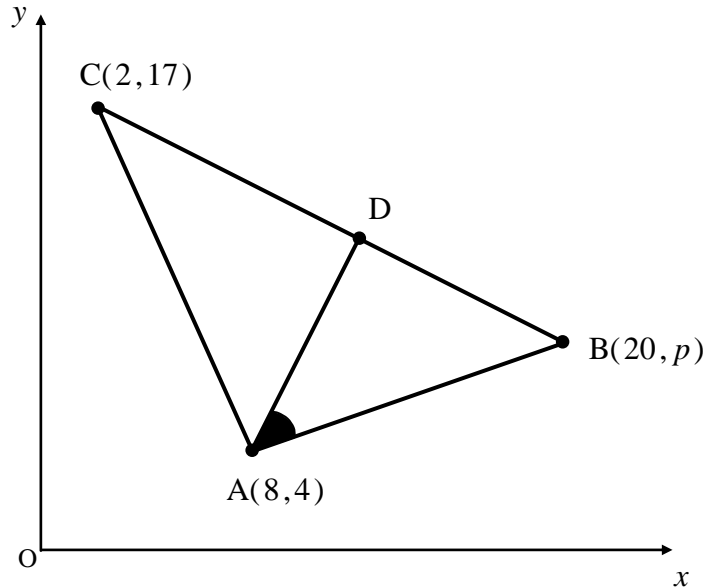
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals:

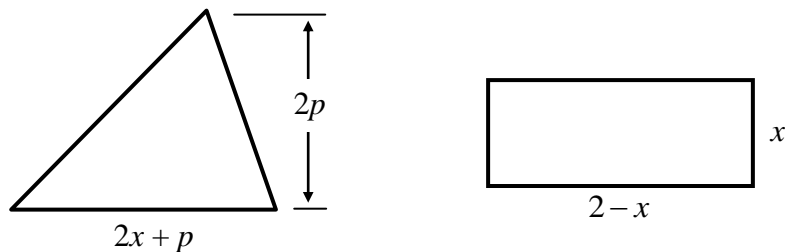
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

**ALL questions should be attempted**

1. In the diagram triangle ABC has vertices  $A(8, 4)$ ,  $B(20, p)$  and  $C(2, 17)$  as shown. **AD is perpendicular to side BC.** [Diagram not drawn to scale]



- (a) Given that the gradient of side BC is  $-\frac{1}{2}$ , find the value of  $p$ . 3
- (b) Find the equation of the altitude AD in the form  $Ax + By + C = 0$ . 3
- (c) By considering the gradients of side AB and the altitude AD, calculate the size of the shaded angle DAB. 3
2. In the diagram below the triangle and the rectangle **have equal areas**.



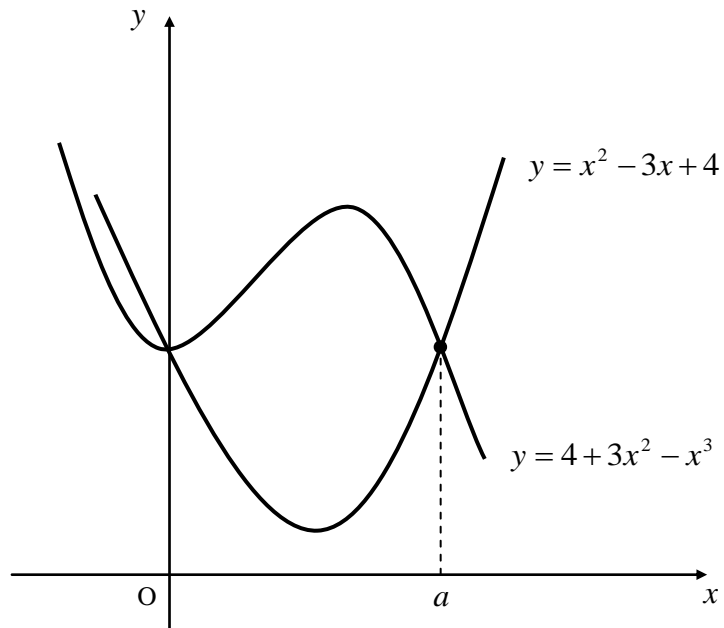
- (a) Show clearly that the following quadratic equation, in  $x$ , can be constructed from the information given.

$$x^2 + (2p - 2)x + p^2 = 0 \quad 4$$

- (b) For what value of  $p$  does the above quadratic equation have equal roots? 4

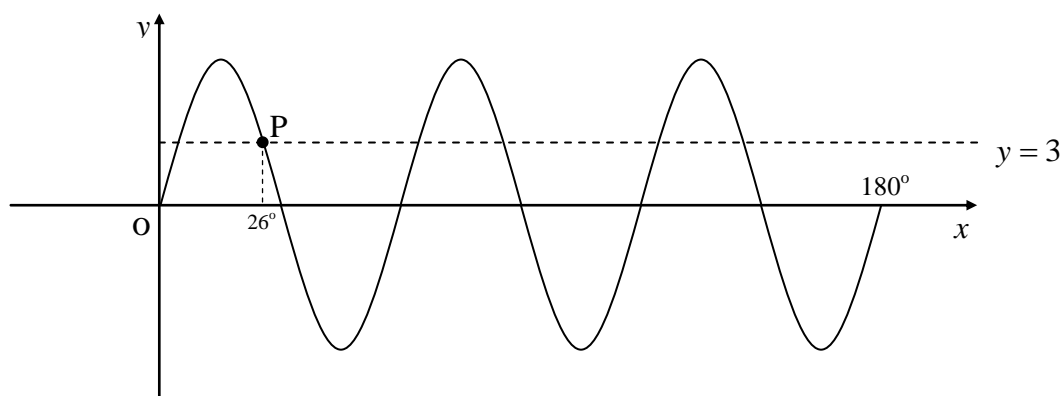
3. The diagram below shows parts of the graphs of  $y = x^2 - 3x + 4$  and  $y = 4 + 3x^2 - x^3$ .

The curves intersect on the  $y$ -axis and at the point with  $x$ -coordinate  $a$ .



- (a) Find, algebraically, the value of  $a$ . 4
- (b) Hence calculate the area enclosed between the two curves. 4

4. Part of the graph of  $y = k \sin nx^\circ$  is shown in the diagram.

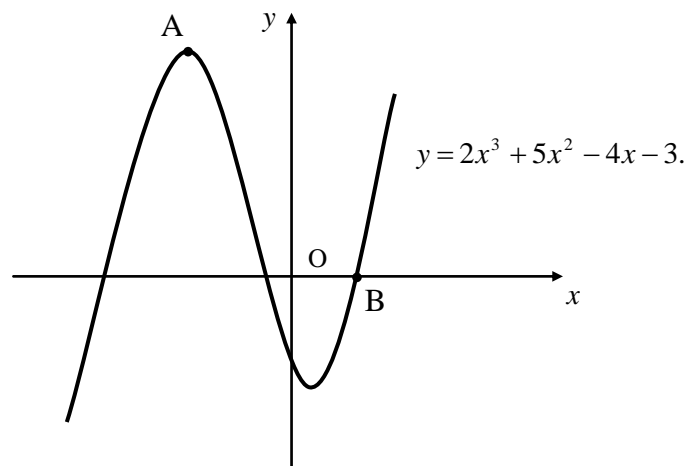


- (a) Write down the value of  $n$ . 1
- The broken line has equation  $y = 3$ .
- (b) Given that the  $x$ -coordinate of the point P is  $26^\circ$ , calculate the value of  $k$  **correct to 1 decimal place**. 3

5. A function is defined on a suitable domain as  $f(x) = \frac{1}{x}(x^2 - \sqrt{x})$ .
- (a) Find the derivative of the function,  $f'(x)$ , expressing your answer with positive indices. 4
- (b) Find the equation of the tangent to the curve  $y = \frac{1}{x}(x^2 - \sqrt{x})$ , at the point where  $x = 1$ . 3

6. A circle has a radius of 10 units and the point  $(-3, 2)$  as its centre.
- (a) Write down the equation of this circle. 1
- (b) Given that the point  $P(5, k)$  lies on this circle, find  $k$  where  $k < 0$ . 4
- (c) Find the equation of the tangent to this circle at the point P. 4
- (d) Show clearly that this tangent passes through the centre of the circle with equation  $x^2 + y^2 - 4x + 16y + 43 = 0$ . 2

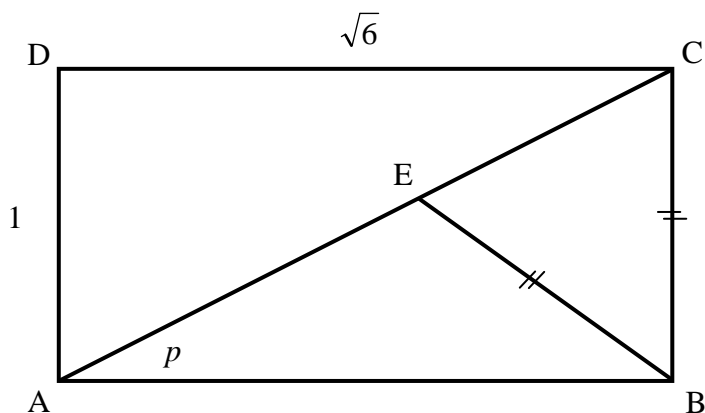
7. The diagram below shows part of the curve with equation  $y = 2x^3 + 5x^2 - 4x - 3$ .



- (a) Find the coordinates of the stationary point marked A. 4
- (b) Find the coordinates of B, one of the points where the curve crosses the  $x$ -axis. 2



8. Rectangle ABCD measures  $\sqrt{6}$  units by 1 unit as shown. The diagram is not drawn to scale. Angle BAC =  $p$  radians. Triangle BCE is isosceles with  $BC = BE$ .



- (a) Show clearly that  $\angle CBE = 2p$ . 3
- (b) Hence calculate the **exact** value of  $\cos \hat{CBE}$ . 4

[ END OF QUESTION PAPER ]

**Paper 1 - Section A - Answer Sheet**

NAME :

CLASS :

TEACHER :

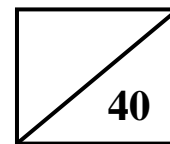
You should use an HB pencil.  
 Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

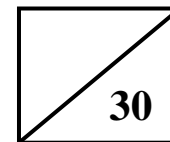
A	B	C	D
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
13	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
14	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
15	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
16	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
17	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
18	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
19	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
20	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

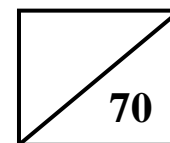
Section A



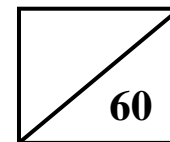
Section B



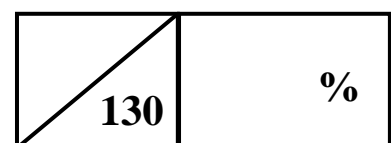
Total (P1)



Total (P2)



Overall Total



*Please make sure you have filled in all your details above before handing in this answer sheet.*

- 1 B
- 2 D
- 3 C
- 4 C
- 5 A
- 6 D
- 7 B
- 8 C
- 9 C
- 10 B
- 11 B
- 12 D
- 13 B
- 14 D
- 15 B
- 16 C
- 17 D
- 18 B
- 19 D
- 20 A

	A	B	C	D
1	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
5	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
7	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
11	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
13	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
14	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
15	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
16	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
17	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
18	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
19	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
20	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	ans: C(9, 7) (1 mark) • <sup>1</sup> states centre of circle	• <sup>1</sup> C(9, 7)
(b)	ans: P(5, 4) (2 marks) • <sup>1</sup> uses a stepping out method or midpoint • <sup>2</sup> finds coordinates of P	• <sup>1</sup> evidence • <sup>2</sup> P(5, 4)
(c)	ans: $(x - 5)^2 + (y - 4)^2 = 25$ (3 marks) • <sup>1</sup> finds radius of circle • <sup>2</sup> starts to write equation of circle • <sup>3</sup> completes equation	• <sup>1</sup> $r = 5$ units • <sup>2</sup> $(x - 5)^2 + (y - 4)^2$ • <sup>3</sup> $(x - 5)^2 + (y - 4)^2 = 25$
22	ans: 0; $x = \frac{1}{2}$ (6 marks) • <sup>1</sup> substitutes $f(x)$ into $g(x)$ • <sup>2</sup> simplifies and multiplies bracket • <sup>3</sup> start to complete square • <sup>4</sup> completes square • <sup>5</sup> states minimum value of function • <sup>6</sup> states corresponding value for $x$	• <sup>1</sup> $g(f(x)) = (2x + 1 - 2)^2$ • <sup>2</sup> $g(f(x)) = (2x - 1)^2 = 4x^2 - 4x + 1$ • <sup>3</sup> $4(x^2 - x) + 1 = 4[(x - \frac{1}{2})^2 - \frac{1}{4}] + 1$ • <sup>4</sup> $4(x - \frac{1}{2})^2 - 1 + 1 = 4(x - \frac{1}{2})^2 =$ • <sup>5</sup> minimum value = 0 • <sup>6</sup> $x = \frac{1}{2}$
23	ans: $A = \frac{\pi}{4}$ or $\frac{\pi}{2}$ (5 marks) • <sup>1</sup> replaces $\sin 2A$ • <sup>2</sup> multiplies and takes common factor • <sup>3</sup> makes each bracket equal to zero and finds first solution • <sup>4</sup> simplifies second bracket • <sup>5</sup> finds solution	• <sup>1</sup> $\sqrt{12}(2 \sin A \cos A) - 2\sqrt{6} \cos A = 0$ • <sup>2</sup> $2 \cos A(\sqrt{12} \sin A - \sqrt{6}) = 0$ • <sup>3</sup> $2 \cos A = 0$ or $(\sqrt{12} \sin A - \sqrt{6}) = 0$ $A = \frac{\pi}{2}$ • <sup>4</sup> $\sin A = \frac{\sqrt{6}}{\sqrt{12}} = \sqrt{\frac{6}{12}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$ • <sup>5</sup> $A = \frac{\pi}{4}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
24(a)	ans: $u_0 = 200$ (2 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes into RR</li> <li>•<sup>2</sup> finds <math>u_0</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>0.8u_0 + 60 = 220</math></li> <li>•<sup>2</sup> <math>u_0 = 200</math></li> </ul>
(b)	ans: 100 (4 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> states condition for limit</li> <li>•<sup>2</sup> knows how to find limit</li> <li>•<sup>3</sup> finds limit</li> <li>•<sup>4</sup> states difference</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> limit exists since <math>-1 &lt; 0.8 &lt; 1</math></li> <li>•<sup>2</sup> <math>L = \frac{60}{1-0.8} = \frac{60}{0.2}</math></li> <li>•<sup>3</sup> 300</li> <li>•<sup>4</sup> 100</li> </ul>
25(a)	ans: $a = 2; b = -2$ (4 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to set up system of equations</li> <li>•<sup>2</sup> uses synthetic division with first root</li> <li>•<sup>3</sup> uses synthetic division with second root</li> <li>•<sup>4</sup> solves for <math>a</math> and <math>b</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> evidence</li> <li>•<sup>2</sup> <math display="block">\begin{array}{r rrrr} 1 &amp; 1 &amp; a &amp; -1 &amp; b \\ &amp; &amp; 1 &amp; 1+a &amp; a \\ \hline &amp; 1 &amp; 1+a &amp; a &amp; a+b=0 \end{array}</math> </li> <li>•<sup>3</sup> <math display="block">\begin{array}{r rrrr} -2 &amp; 1 &amp; a &amp; -1 &amp; b \\ &amp; &amp; -2 &amp; 4-2a &amp; 4a-6 \\ \hline &amp; 1 &amp; a-2 &amp; 3-2a &amp; 4a+b=6 \end{array}</math> </li> <li>•<sup>4</sup> <math>a = 2; b = -2</math></li> </ul>
(b)	ans: $x = -1$ (3 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes for <math>a</math> and <math>b</math> and uses syth.div.</li> <li>•<sup>2</sup> factorises</li> <li>•<sup>3</sup> states third factor</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math display="block">\begin{array}{r rrrr} 1 &amp; 1 &amp; 2 &amp; -1 &amp; -2 \\ &amp; &amp; 1 &amp; 3 &amp; 2 \\ \hline &amp; 1 &amp; 3 &amp; 2 &amp; 0 \end{array}</math> </li> <li>•<sup>2</sup> <math>(x-1)(x+2)(x+1) = 0</math></li> <li>•<sup>3</sup> <math>x = -1</math></li> </ul>
		Total: 70 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $p = 8$ (3 marks)  <ul style="list-style-type: none"> <li>•<sup>1</sup> finds expressions for gradient of BC</li> <li>•<sup>2</sup> equates to <math>-\frac{1}{2}</math></li> <li>•<sup>3</sup> solves for <math>p</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>m_{BC} = \frac{p-17}{20-2} = \frac{p-17}{18}</math></li> <li>•<sup>2</sup> <math>\frac{p-17}{18} = -\frac{1}{2}</math></li> <li>•<sup>3</sup> <math>p = 8</math></li> </ul>
(b)	ans: $y - 2x + 12 = 0$ (3 marks)  <ul style="list-style-type: none"> <li>•<sup>1</sup> states gradient of AD</li> <li>•<sup>2</sup> substitutes into <math>y - b = m(x - a)</math></li> <li>•<sup>3</sup> rearranges into required form</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>m_{AD} = 2</math></li> <li>•<sup>2</sup> <math>y - 4 = 2(x - 8)</math></li> <li>•<sup>3</sup> <math>y - 2x + 12 = 0</math></li> </ul>
(c)	ans: $45^\circ$ (3 marks)  <ul style="list-style-type: none"> <li>•<sup>1</sup> finds gradient of AB</li> <li>•<sup>2</sup> takes <math>\tan^{-1}</math> of both gradients</li> <li>•<sup>3</sup> subtracts to find angle DAB</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>m_{AB} = \frac{8-4}{20-8} = \frac{1}{3}</math></li> <li>•<sup>2</sup> <math>\tan^{-1}\left(\frac{1}{3}\right) = 18 \cdot 4^\circ; \tan^{-1}(2) = 63 \cdot 4^\circ</math></li> <li>•<sup>3</sup> <math>45^\circ</math></li> </ul>
2(a)	ans: proof (4 marks)  <ul style="list-style-type: none"> <li>•<sup>1</sup> finds expressions for both areas</li> <li>•<sup>2</sup> equates expressions and starts to simplify</li> <li>•<sup>3</sup> brings all terms to LHS</li> <li>•<sup>4</sup> rearranges to required form</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>p(2x + p)</math> and <math>x(2 - x)</math></li> <li>•<sup>2</sup> <math>2xp + p^2 = 2x - x^2</math></li> <li>•<sup>3</sup> <math>2xp + p^2 - 2x + x^2 = x^2 - 2x + 2xp + p^2</math></li> <li>•<sup>4</sup> <math>x^2 + (2p - 2)x + p^2 = 0</math></li> </ul>
(b)	ans: $p = \frac{1}{2}$ (4 marks)  <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to make discriminant equal to 0</li> <li>•<sup>2</sup> lists values of <math>a</math>, <math>b</math> and <math>c</math></li> <li>•<sup>3</sup> substitutes and simplifies</li> <li>•<sup>4</sup> solves for <math>p</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>b^2 - 4ac = 0</math> [stated or implied]</li> <li>•<sup>2</sup> <math>a = 1; b = 2p - 2; c = p^2</math></li> <li>•<sup>3</sup> <math>(2p - 2)^2 - 4 \cdot 1 \cdot p^2 = 0; 4p^2 - 8p + 4 - 4p^2 = 0</math></li> <li>•<sup>4</sup> <math>p = \frac{1}{2}</math></li> </ul>

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	<b>ans: <math>a = 3</math> (4 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to equate the two functions</li> <li>•<sup>2</sup> brings all terms to LHS and simplify</li> <li>•<sup>3</sup> factorises</li> <li>•<sup>4</sup> solves for <math>a</math> and chooses correct value of <math>a</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>a^2 - 3a + 4 = 4 + 3a^2 - a^3</math></li> <li>•<sup>2</sup> <math>a^3 - 2a^2 - 3a = 0</math></li> <li>•<sup>3</sup> <math>a(a^2 - 2a - 3) = 0; a(a - 3)(a + 1) = 0</math></li> <li>•<sup>4</sup> <math>a = 0, 3, -1; a = 3</math></li> </ul>
(b)	<b>ans: <math>11.25 \text{ units}^2</math> (4 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> sets up integral</li> <li>•<sup>2</sup> integrates</li> <li>•<sup>3</sup> substitutes limits</li> <li>•<sup>4</sup> evaluates</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int_0^3 4 + 3x^2 - x^3 - (x^2 - 3x + 4) dx</math>  <math>= \int_0^3 2x^2 - x^3 + 3x dx</math></li> <li>•<sup>2</sup> <math>\left[ \frac{2x^3}{3} - \frac{x^4}{4} + \frac{3x^2}{2} \right]_0^3</math></li> <li>•<sup>3</sup> <math>\left[ \frac{2(3)^3}{3} - \frac{(3)^4}{4} + \frac{3(3)^2}{2} \right]_0^3</math></li> <li>•<sup>4</sup> <math>11.25 \text{ units}^2</math></li> </ul>
4(a)	<b>ans: <math>n = 6</math> (1 mark)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> states value of <math>n</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>n = 6</math></li> </ul>
(b)	<b>ans: <math>k = 7.4</math> (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes and equates</li> <li>•<sup>2</sup> finds expression for <math>k</math></li> <li>•<sup>3</sup> value of <math>k</math> correctly rounded</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k \sin(6 \times 26) = 3</math></li> <li>•<sup>2</sup> <math>k = \frac{3}{\sin 156^\circ}</math></li> <li>•<sup>3</sup> <math>k = 7.4</math></li> </ul>
5(a)	<b>ans: <math>1 + \frac{1}{2x^{\frac{3}{2}}}</math> (4 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> starts to prepare to differentiate</li> <li>•<sup>2</sup> completes preparation</li> <li>•<sup>3</sup> differentiates both terms</li> <li>•<sup>4</sup> writes with positive indices</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^{-1}(x^2 - x^{\frac{1}{2}})</math></li> <li>•<sup>2</sup> <math>x - x^{-\frac{1}{2}}</math></li> <li>•<sup>3</sup> <math>1 + \frac{1}{2}x^{-\frac{3}{2}}</math></li> <li>•<sup>4</sup> <math>1 + \frac{1}{2x^{\frac{3}{2}}}</math></li> </ul>
(b)	<b>ans: <math>2y = 3x - 3</math> (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes into function for point</li> <li>•<sup>2</sup> substitutes into derivative for gradient</li> <li>•<sup>3</sup> states equations of tangent</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(1, 0)</math></li> <li>•<sup>2</sup> <math>m = 3/2</math></li> <li>•<sup>3</sup> <math>y - 0 = 3/2(x - 1); 2y = 3x - 3</math></li> </ul>

	Give 1 mark for each •	Illustration(s) for awarding each mark
6(a)	<b>ans: <math>(x + 3)^2 + (y - 2)^2 = 100</math> (1 mark)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> subs into equation of circle</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(x + 3)^2 + (y - 2)^2 = 100</math></li> </ul>
(b)	<b>ans: <math>k = -4</math> (4 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes point into circle equation</li> <li>•<sup>2</sup> simplifies</li> <li>•<sup>3</sup> solves for <math>k</math></li> <li>•<sup>4</sup> chooses appropriate value for <math>k</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(5 + 3)^2 + (k - 2)^2 = 100</math></li> <li>•<sup>2</sup> <math>64 + (k - 2)^2 = 100; (k - 2)^2 = 36</math></li> <li>•<sup>3</sup> <math>k - 2 = \pm 6; k = 8</math> or <math>k = -4</math></li> <li>•<sup>4</sup> <math>k = -4</math></li> </ul>
(c)	<b>ans: <math>3y - 4x + 32 = 0</math> (4 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> finds gradient of CP</li> <li>•<sup>2</sup> finds perpendicular gradient</li> <li>•<sup>3</sup> substitutes into equation</li> <li>•<sup>4</sup> simplifies</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>m_{CP} = -\frac{3}{4}</math></li> <li>•<sup>2</sup> <math>m_{\tan} = \frac{4}{3}</math></li> <li>•<sup>3</sup> <math>y + 4 = \frac{4}{3}(x - 5)</math></li> <li>•<sup>4</sup> <math>3y - 4x + 32 = 0</math> [or equivalent]</li> </ul>
(d)	<b>ans: proof (2 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> finds centre of circle</li> <li>•<sup>2</sup> substitutes into equation of tangent</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> centre of circle is <math>(2, -8)</math></li> <li>•<sup>2</sup> <math>3(-8) - 4(2) + 32 = 0</math> which is correct tangent passes through centre of circle</li> </ul>
7(a)	<b>ans: A(-2, 9) (4 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> knows <math>\frac{dy}{dx} = 0</math></li> <li>•<sup>2</sup> differentiates and equates to zero</li> <li>•<sup>3</sup> solves for <math>x</math> and chooses required value</li> <li>•<sup>4</sup> finds coordinates of point A</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> evidence stated or implied</li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 6x^2 + 10x - 4 = 0</math> at SP</li> <li>•<sup>3</sup> <math>2(3x - 1)(x + 2) = 0; x = -2</math> or <math>1/3</math></li> <li>•<sup>4</sup> A(-2, 9)</li> </ul>
(b)	<b>ans: B(1, 0) (2 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> makes <math>y = 0</math> and uses synthetic division</li> <li>•<sup>2</sup> factorises, solves and chooses approp. value states coordinates of B</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math display="block">\begin{array}{r rrrr} 1 &amp; 2 &amp; 5 &amp; -4 &amp; -3 \\ &amp; &amp; 2 &amp; 7 &amp; 3 \\ \hline &amp; 2 &amp; 7 &amp; 3 &amp; 0 \end{array}</math> </li> <li>•<sup>2</sup> <math>(x - 1)(2x + 1)(x + 3) = 0</math> B(1, 0)</li> </ul>



	Give 1 mark for each •	Illustration(s) for awarding each mark
8(a)	<p>ans: proof (3 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> finds expression for <math>\angle ACB</math></li> <li>•<sup>2</sup> knows <math>\angle CEB = \angle ACB</math></li> <li>•<sup>3</sup> finds expression for <math>\angle CBE</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\angle ACB = 90 - p</math></li> <li>•<sup>2</sup> <math>\angle CEB = 90 - p</math></li> <li>•<sup>3</sup> <math>\angle CBE = 180 - 2(90 - p) = 2p</math></li> </ul>
(b)	<p>ans: <math>\frac{5}{7}</math> (4 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> uses expansion for <math>\cos 2p</math></li> <li>•<sup>2</sup> finds value of <math>\cos p</math> and <math>\sin p</math></li> <li>•<sup>3</sup> subs into expansion</li> <li>•<sup>4</sup> simplifies to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\cos 2p = \cos^2 p - \sin^2 p</math></li> <li>•<sup>2</sup> <math>\cos p = \frac{\sqrt{6}}{\sqrt{7}}; \sin p = \frac{1}{\sqrt{7}}</math></li> <li>•<sup>3</sup> <math>\cos 2p = \left(\frac{\sqrt{6}}{\sqrt{7}}\right)^2 - \left(\frac{1}{\sqrt{7}}\right)^2</math></li> <li>•<sup>4</sup> <math>\frac{6}{7} - \frac{1}{7} = \frac{5}{7}</math></li> </ul>
		<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Total: 60 marks</div>