

# PRELIM 2008-09

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INVERGORDON ACADEMY  
MATHEMATICS DEPARTMENT

**MATHEMATICS**  
**HIGHER**  
Units 1 and 2  
Paper 1  
(Non-calculator)

TIME: 1 hour 30 minutes

**Read carefully**

**Calculators may NOT be used in this paper.**

**Section A - Questions 1 - 20 (40 marks)**

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB** pencil.

**Section B (30 marks)**

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.



## Read carefully

- 1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2008/2009 (Section A)**.
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- 3 Make sure you write your **name, class and teacher** on the answer sheet provided.
- 4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
- 5 There is **only one correct** answer to each question.
- 6 Rough working should **not** be done on your answer sheet.
- 7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

## Sample Question

A line has equation  $y = 4x - 1$ .

If the point  $(k, 7)$  lies on this line, the value of  $k$  is

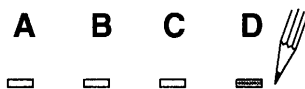
- A 2
- B 27
- C 1.5
- D -2

The correct answer is **A**  $\rightarrow$  2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



## Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**SECTION A**

**ALL questions should be attempted**

1. If  $f(x) = 2x^{\frac{3}{2}}$  then  $f'(4)$  equals
- A 16  
B 4  
C  $25\frac{3}{5}$   
D 6
2. Triangle ABC has vertices A(-3, -3), B(12, -1) and C(6, 11).  
The gradient of the **altitude** through B is
- A  $-\frac{9}{14}$   
B  $\frac{14}{9}$   
C  $-\frac{3}{8}$   
D  $\frac{8}{3}$
3. The remainder when  $x^3 - 11x + 10$  is divided by  $(x + 3)$  is
- A 52  
B 16  
C 4  
D -24
4. The point P(8, y) lies on the circle with equation  $x^2 + y^2 - 12x + 4y + 20 = 0$ .  
The value(s) of y is/are
- A 2 only  
B -6 only  
C -6 and 2  
D 6 and -2

5. A sequence is defined by the recurrence relation  $U_{n+1} = aU_n - 5$  with  $U_0 = 10$ .

An expression in terms of  $a$  for  $U_2$  is

- A  $10a - 5$
- B  $10a^2 - 5$
- C  $10a^2 - 5a - 5$
- D  $10a^2$

6.  $\int_0^1 4x(x^2 - 2) dx$  is

- A  $-3$
- B  $-4$
- C  $0$
- D  $12$

7. The equation  $2x^2 + 8 = kx$  has **no real roots**.  $k$  must take the values

- A  $\pm 8$
- B  $-8 < k < 8$
- C  $k < -8$  or  $k > 8$
- D undefined

8. For which value(s) of  $x$  is the function  $f(x) = \frac{3}{(x+3)(x-2)}$  undefined?

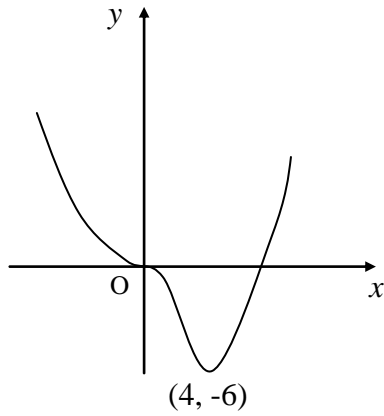
- A  $3$
- B  $3$  and  $-2$
- C  $-3$  and  $2$
- D  $-6$

9. The line  $ax - 2y + 5 = 0$  is parallel to the line with equation  $3x + y - 4 = 0$ .

The value of  $a$  is

- A  $-3$
- B  $-6$
- C  $\frac{2}{3}$
- D  $-\frac{3}{2}$

10.

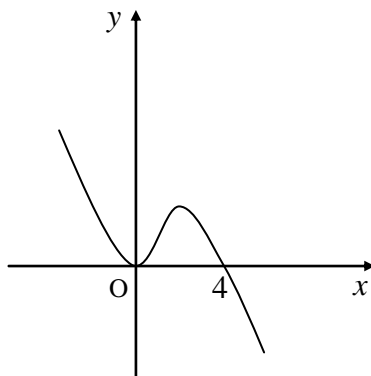


The diagram shows part of the graph of  $y = f(x)$ .

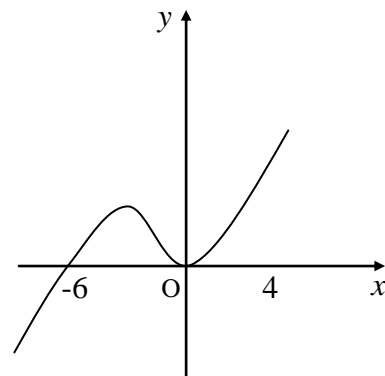
It has stationary points at  $(0, 0)$  and  $(4, -6)$ .

Which of the following could be part of the graph of the derived function  $y = f'(x)$ ?

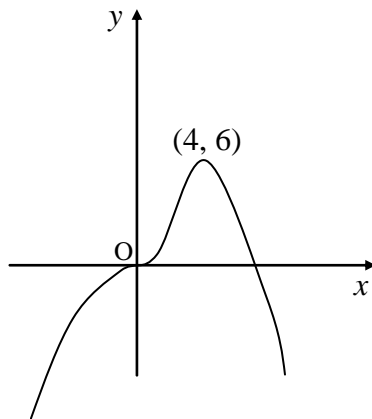
A



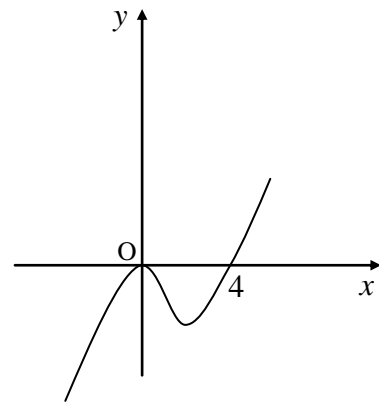
B



C



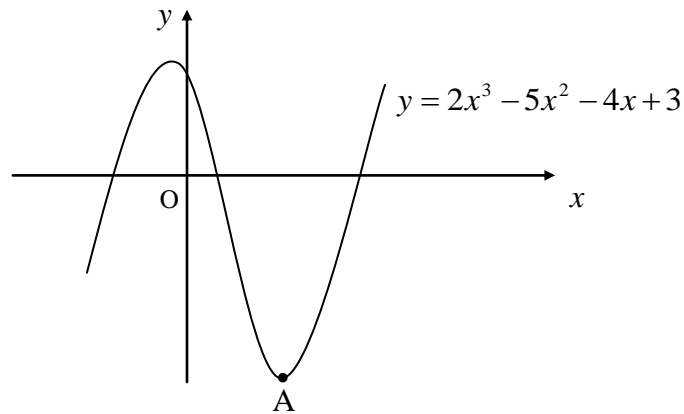
D



11. The two sequences defined by the recurrence relations  $U_{n+1} = 0.5U_n + 20$  and  $V_{n+1} = 0.2V_n + k$  have the same limit. The value of  $k$  is

- A 8
- B 20
- C 40
- D 32

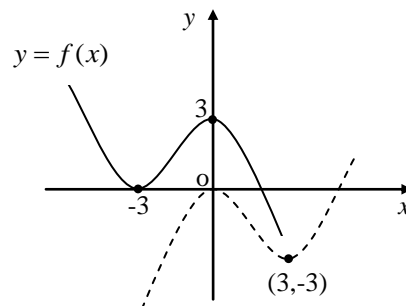
12. The diagram shows part of the curve with equation  $y = 2x^3 - 5x^2 - 4x + 3$ .



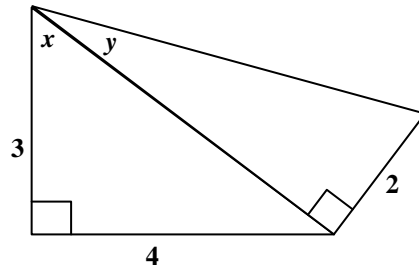
The  $x$ - coordinate of the point A is

- A  $\frac{1}{3}$   
 B 2  
 C 3  
 D -9
13. The function  $f$  is defined as  $f(x) = \frac{x-6}{x}$ ,  $x \neq 0$ . The value of  $f(f(3))$  equals
- A 7  
 B -7  
 C -5  
 D -1
14. The diagram shows the graph of  $y = f(x)$  as a full line and the graph of a related function as a broken line. The equation of the related function is

- A  $y = -f(x) - 3$   
 B  $y = f(x-6) - 3$   
 C  $y = f(-x) - 3$   
 D  $y = f'(x)$



15. The diagram shows two right-angled triangles with lengths as shown.



The exact value of  $\cos(x + y)$  is

- A  $\frac{8}{\sqrt{29}}$
- B  $\frac{7}{5\sqrt{29}}$
- C  $\frac{23}{5\sqrt{29}}$
- D  $\frac{3}{\sqrt{29}}$
16. A circle has centre A(1, 3) and radius  $\sqrt{5}$ . Another circle has centre B(9, 7) and radius  $3\sqrt{5}$ . Which of the following is true for these two circles?
- A they intersect at two points
- B they touch externally
- C they touch internally
- D they do not intersect or touch
17. The maximum value of  $\frac{12}{x^2 - 4x + 10}$  is
- A 2
- B -2
- C 6
- D -6



18. A ball is thrown upwards reaching a height of ' $h$ ' metres after ' $t$ ' seconds where  $h(t) = 2 + 12t - 3t^2$ . The time taken, in seconds, to reach its maximum height is

- A 2
- B 3
- C 4
- D 5

19. The exact value of  $\sin \frac{2\pi}{3} - \cos \frac{7\pi}{6}$  is

- A 0
- B 1
- C  $\sqrt{3}$
- D  $\frac{\sqrt{3}}{4}$

20.  $(x, y)$  is a solution for the system of equations

$$x^2 + 7y^2 = 16$$

$$x - 3y = 0.$$

Possible values for  $x + y$  are

(1) 0    (2) 4    (3) -4

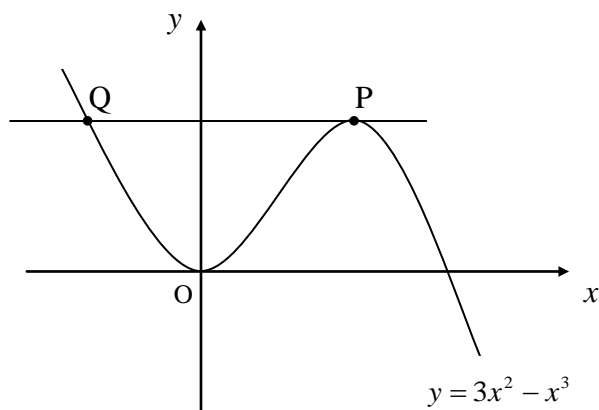
- A (1) only
- B (2) only
- C (2) and (3) only
- D (1), (2) and (3)

[ END OF SECTION A ]

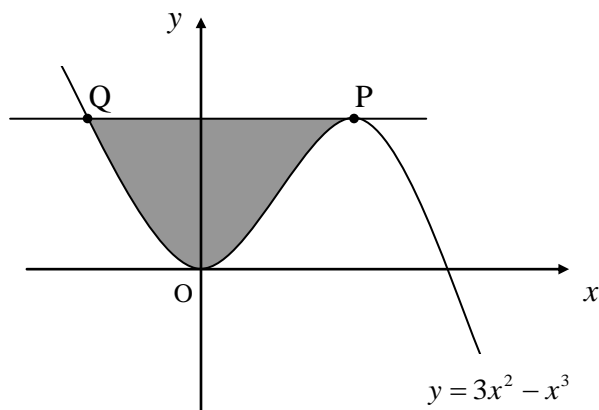
**SECTION B**

**ALL questions should be attempted**

- 21.** Part of the graph of the curve with equation  $y = 3x^2 - x^3$  is shown below. The diagram is not drawn to scale.



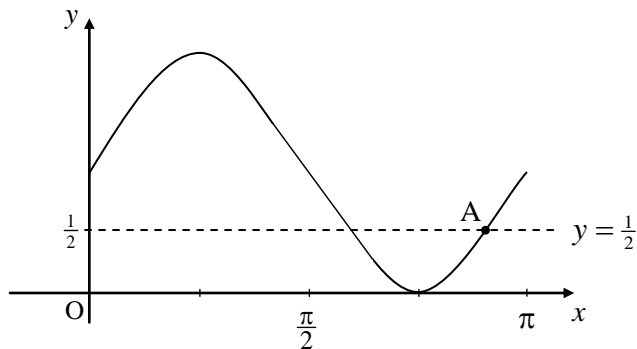
- (a) Establish the coordinates of the stationary point P. **4**
- (b) The horizontal line through P meets the curve again at Q. Find the coordinates of Q. **3**
- (c) Hence calculate the shaded area shown in the diagram below.



**6**

- 22.** Two functions, defined on suitable domains, are given as  $f(x) = x^2 - 1$  and  $g(x) = 2 - x$ .
- (a) Show that  $f(g(a))$  can be expressed in the form  $pa^2 + qa + r$  and write down the values of  $p$ ,  $q$  and  $r$ . **4**
- (b) Hence find  $a$  if  $f(g(a)) = 8$  and  $a > 0$ . **2**

23. The diagram below shows part of the graph of  $y = \sin 2x + 1$ , for  $0 \leq x \leq \pi$ , and the line with equation  $y = \frac{1}{2}$ .



Find the coordinates of the point A.

4

24. A recurrence relation is defined by the formula  $U_{n+1} = 0.6U_n + 24$ .

(a) Establish the limit of this sequence.

2

(b) Given now that  $U_1$  is exactly half of this limit, find  $U_0$ , the initial value of the sequence.

2

(c) A second recurrence relation in the form  $U_{n+1} = aU_n + b$  has the **same limit** as the sequence above and is such that  $b = 90a$ .

Find the values of  $a$  and  $b$  in this second sequence.

3

[ END OF SECTION B ]

[ END OF QUESTION PAPER ]

**Mathematics**  
**Higher Prelim Examination 2008/2009**

**NATIONAL  
 QUALIFICATIONS**

**Paper 1 - Section A - Answer Sheet**

NAME :

CLASS :

TEACHER :

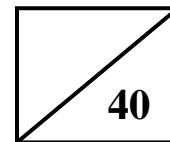
You should use an HB pencil.  
 Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

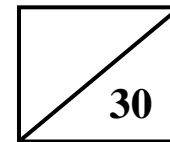
A	B	C	D
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	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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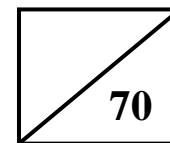
Section A



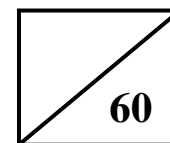
Section B



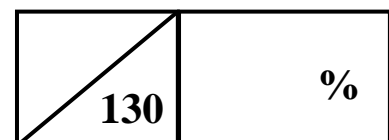
Total (P1)



Total (P2)



Overall Total



*Please make sure you have filled in all your details above before handing in this answer sheet.*

Higher Grade - Paper 1 2008/2009

ANSWERS - Section A

- 1 D
- 2 A
- 3 B
- 4 C
- 5 C
- 6 A
- 7 B
- 8 C
- 9 B
- 10 D
- 11 D
- 12 B
- 13 A
- 14 C
- 15 B
- 16 B
- 17 A
- 18 A
- 19 C
- 20 C

	A	B	C	D
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20	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	ans: P(2, 4) (4 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to make derivative equal to 0</li> <li>•<sup>2</sup> finds derivative</li> <li>•<sup>3</sup> solves for x</li> <li>•<sup>4</sup> states coordinates of P</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 0</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 6x - 3x^2 = 0</math></li> <li>•<sup>3</sup> <math>3x(2 - x) = 0; x = 0, 2</math></li> <li>•<sup>4</sup> P(2, 4)</li> </ul>
	(b) ans: Q(-1, 4) (3 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to equate functions</li> <li>•<sup>2</sup> uses app method to factorise expression</li> <li>•<sup>3</sup> solves and states coordinates of Q</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>3x^2 - x^3 = 4</math></li> <li>•<sup>2</sup> evidence leading to <math>(x - 2)(x - 2)(x + 1)</math></li> <li>•<sup>3</sup> Q(-1, 4)</li> </ul>
(c)	ans: $6\frac{3}{4}$ units <sup>2</sup> (6 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to use integration</li> <li>•<sup>2</sup> uses correct integration</li> <li>•<sup>3</sup> integrates</li> <li>•<sup>4</sup> subs values</li> <li>•<sup>5</sup> evaluates</li> <li>•<sup>6</sup> evaluates to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int \dots\dots</math></li> <li>•<sup>2</sup> <math>\int_{-1}^2 (4 - (3x^2 - x^3))dx</math></li> <li>•<sup>3</sup> <math>\left[4x - x^3 + \frac{x^4}{4}\right]_{-1}^2</math></li> <li>•<sup>4</sup> <math>\left[4(2) - (2)^3 + \frac{(2)^4}{4}\right] - \left[4(-1) - (-1)^3 + \frac{(-1)^4}{4}\right]</math></li> <li>•<sup>5</sup> <math>(8 - 8 + 4) - (-4 + 1 + \frac{1}{4})</math></li> <li>•<sup>6</sup> <math>4 + 2\frac{3}{4} = 6\frac{3}{4}</math> units<sup>2</sup></li> </ul>
22(a)	ans: p = 1, q = -4, r = 3 (4 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes</li> <li>•<sup>2</sup> multiplies out and reorganises</li> <li>•<sup>3</sup> states values of p, q and r</li> <li>•<sup>4</sup> states values of p, q and r</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(g(a)) = (2 - a)^2 - 1</math></li> <li>•<sup>2</sup> <math>4 - 4a + a^2 - 1 = a^2 - 4a + 3</math></li> <li>•<sup>3</sup> <math>p = 1, q = -4, r = 3</math></li> <li>•<sup>4</sup> award 1 mark for any two correct and 1 mark for third value correct</li> </ul>
(b)	ans: a = 5 (2 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> equates to 8, reorganises and factorises</li> <li>•<sup>2</sup> solves and chooses correct value for a</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>a^2 - 4a + 3 = 8; a^2 - 4a - 5 = 0</math> <math>(a - 5)(a + 1) = 0</math></li> <li>•<sup>2</sup> <math>a = 5</math></li> </ul>

	Give 1 mark for each •	Illustration(s) for awarding each mark
23	<p>ans: <math>A\left(\frac{11\pi}{12}, \frac{1}{2}\right)</math> (4 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> equates line &amp; curve, reorganises</li> <li>•<sup>2</sup> finds values for <math>2x</math></li> <li>•<sup>3</sup> finds values for <math>x</math></li> <li>•<sup>4</sup> states coordinates of A</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\sin 2x + 1 = \frac{1}{2}; \sin 2x = -\frac{1}{2}</math></li> <li>•<sup>2</sup> <math>2x = \frac{7\pi}{6}, \frac{11\pi}{6}</math></li> <li>•<sup>3</sup> <math>x = \frac{7\pi}{12}, \frac{11\pi}{12}</math></li> <li>•<sup>4</sup> <math>A\left(\frac{11\pi}{12}, \frac{1}{2}\right)</math></li> </ul>
24(a)	<p>ans: 60 (2 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> knows how to find limit</li> <li>•<sup>2</sup> moves term to LHS and divides</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>L = \frac{24}{1 - 0.6}</math></li> <li>•<sup>2</sup> 60</li> </ul>
(b)	<p>ans: 10 (2 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> makes RR equal to 30</li> <li>•<sup>2</sup> solves for <math>U_0</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>0.6U_0 + 24 = 30</math> [stated or implied]</li> <li>•<sup>2</sup> <math>0.6U_0 = 6; U_0 = 10</math></li> </ul>
(c)	<p>ans: <math>a = \frac{2}{5}; b = 36</math> (3 marks)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> subs for <math>b</math> and finds expression for limit</li> <li>•<sup>2</sup> equates limit to 60 and solves for <math>a</math></li> <li>•<sup>3</sup> finds value of <math>b</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>U_{n+1} = aU_n + 90a; L = \frac{90a}{1-a}</math></li> <li>•<sup>2</sup> <math>\frac{90a}{1-a} = 60; 60 - 60a = 90a; a = \frac{2}{5};</math></li> <li>•<sup>3</sup> <math>b = 90 \times \frac{2}{5} = 36</math></li> </ul>

# PRELIM 2007-08

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INVERGORDON ACADEMY  
MATHEMATICS DEPARTMENT

MATHEMATICS  
HIGHER  
Units 1 and 2  
Paper 2

TIME: 1 hour 10 minutes

## Read carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.





## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

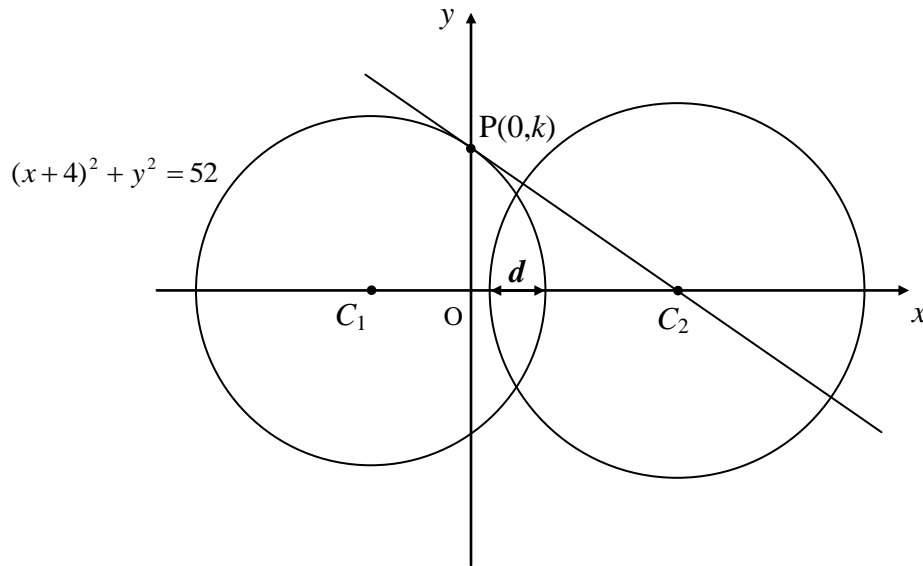
**ALL questions should be attempted**

1. Consider the diagram below.

The circle centre  $C_1$  has as its equation  $(x+4)^2 + y^2 = 52$ .

The point  $P(0, k)$  lies on the circumference of this circle and the tangent to this circle through  $P$  has been drawn.

A second circle with centre  $C_2$  is also shown.



- (a) What is the value of  $k$ ? 2
- (b) Hence find the equation of the tangent through  $P$ . 4
- (c) The tangent through  $P$  passes through  $C_2$  the centre of the second circle. State the coordinates of  $C_2$ . 1
- (d) Given that the second circle has a radius of 8 units, calculate the distance marked  $d$  on the diagram, giving your answer correct to 1 decimal place. 3

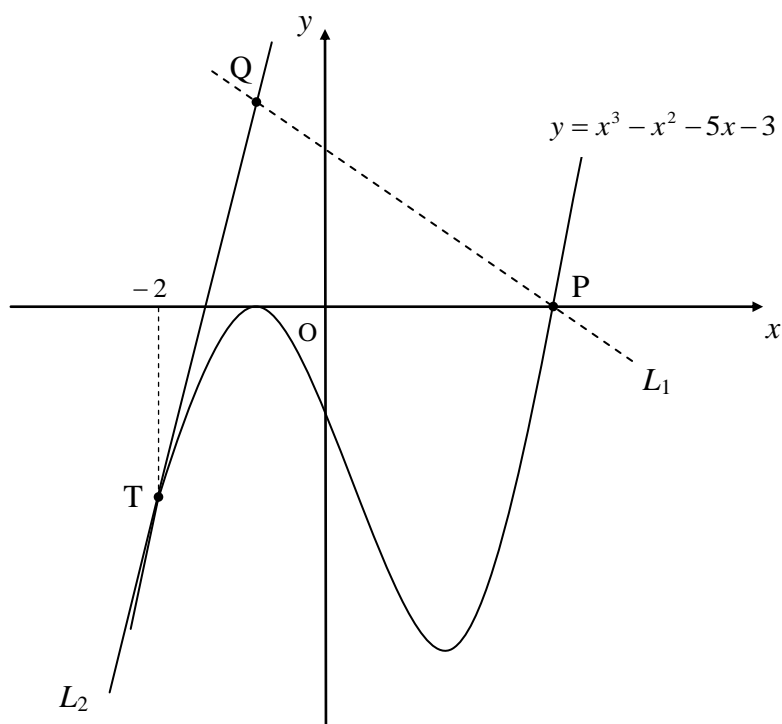
2. Solve algebraically the equation

$$3\cos 2x^\circ + 4\sin x^\circ - 1 = 0 \quad \text{for} \quad 0 \leq x < 360. \quad 5$$

3. A curve has as its derivative  $\frac{dy}{dx} = 2x - \frac{6}{x^2}$ .
- (a) Given that the point (2, 3) lies on this curve, express  $y$  in terms of  $x$ . 4
- (b) Hence find  $p$  if the point (3,  $p$ ) also lies on this curve. 1

4. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve with equation  $y = x^3 - x^2 - 5x - 3$ .

Two straight lines are also shown,  $L_1$  and  $L_2$ .



- (a) Find the coordinates of P. 2
- (b) Line  $L_1$  has a gradient of  $-\frac{3}{2}$  and passes through the point P. Find the equation of  $L_1$ . 1
- (c) Line  $L_2$  is a tangent to the curve at the point T where  $x = -2$ . Find the equation of  $L_2$ . 4
- (d) Hence find the coordinates of Q, the point of intersection of the two lines. 3

5. A company making commercial "glow sticks" have devised a method to test the brightness and consistency of the glow given off.

All glow sticks depend on a chemical process known as chemiluminescence to produce their light. Once a glow stick has been illuminated (by mixing two chemicals together) the brightness of the glow diminishes over a period of time.



**When one of their glow sticks is ignited the initial brightness is rated at 200 *gu* (glow units).**

- (a) During any 1 hour period the glow light is known to lose 8% of its brightness at the beginning of the period.

Calculate the brightness remaining, in *gu*'s, after a period of 4 hours.

**2**

- (b) At the end of each 4 hour period, the glow light is automatically passed through a tube which has an internal temperature of  $-40^{\circ}C$ . This lowering of the temperature of the glow light has the effect of allowing it to regain some of its lost brightness. A single pass through this refrigerated tube allows the glow stick to regain 32 glow units.

The 4 hour cycle described above is now left to run uninterrupted for a total of 16 hours.

By considering an appropriate recurrence relation, calculate the brightness remaining, in *gu*'s, after this 16 hour period has been completed.

**Your answer must be accompanied with the appropriate working.**

**3**

- (c) If this cycle was left to run over a very long period of time would the brightness of the glow stick ever drop to below half of its initial brightness? Explain your answer.

**Your answer and explanation must be accompanied with the appropriate working.**

**3**

6. (a) If  $k = \frac{(x-1)^2}{x^2+4}$ , where  $k$  is a real number, show clearly that

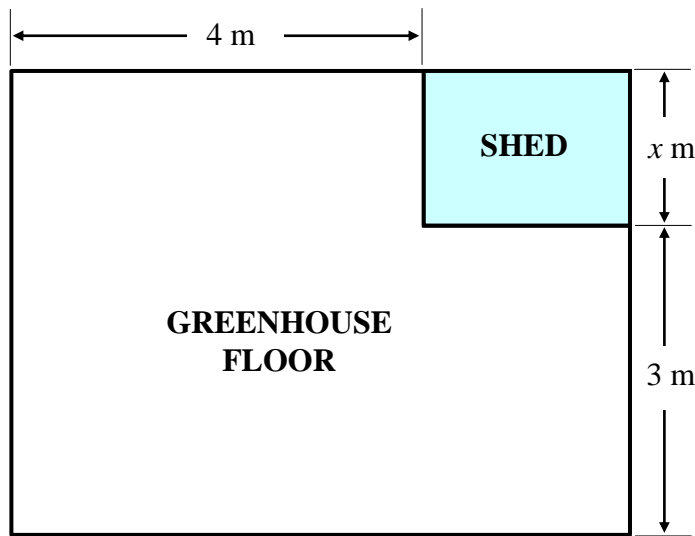
$$(k-1)x^2 + 2x + (4k-1) = 0.$$

**3**

- (b) Hence find the value of  $k$  given that the equation  $(k-1)x^2 + 2x + (4k-1) = 0$  has equal roots and  $k > 0$ .

**5**

7. The floor plan of a rectangular greenhouse is shown below. All dimensions are in metres. The gardener places a rectangular wooden storage shed, of width  $x$  metres, in one corner.



- (a) Given that the **area of the shed** is 3 square metres, show clearly that the area of greenhouse floor remaining,  $A$  square metres, is given in terms of  $x$  as

$$A(x) = 12 + 4x + \frac{9}{x} . \quad 3$$

- (b) Hence find the value of  $x$  which **minimises** the area of the greenhouse floor remaining, **justifying your answer**. 5

8. Angle  $A$  is acute and such that  $\tan A = \frac{\sqrt{6}}{3}$ .

- (a) Show clearly that the exact value of  $\sin A$  can be written in the form  $\frac{1}{5}\sqrt{k}$ , and state the value of  $k$ . 3

- (b) Hence, or otherwise, show that the value of  $\cos 2A$  is exactly  $\frac{1}{5}$  3

[ END OF QUESTION PAPER ]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $k = 6$ (2 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to substitute point</li> <li>•<sup>2</sup> establishes value of <math>k</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(0+4)^2 + k^2 = 52</math></li> <li>•<sup>2</sup> <math>k = 6</math></li> </ul>
(b)	ans: $y = -\frac{2}{3}x + 6$ (4 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> finds coordinates of <math>C_1</math></li> <li>•<sup>2</sup> finds gradient of radius</li> <li>•<sup>3</sup> finds gradient of tangent</li> <li>•<sup>4</sup> substitutes into formula</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>C(-4, 0)</math></li> <li>•<sup>2</sup> <math>m_{C_1P} = \frac{6}{4} = \frac{3}{2}</math></li> <li>•<sup>3</sup> <math>m_{\tan} -\frac{2}{3}</math></li> <li>•<sup>4</sup> <math>y = -\frac{2}{3}x + 6</math></li> </ul>
(c)	ans: $C_2(9, 0)$ (1 mark) <ul style="list-style-type: none"> <li>•<sup>1</sup> subs point, solves for <math>x</math> and states point</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>0 = -\frac{2}{3}x + 6; x = 9; (9, 0)</math></li> </ul>
(d)	ans: 2.2 units (3 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> finds radius <math>C_1</math> circle</li> <li>•<sup>2</sup> finds distance between centres</li> <li>•<sup>3</sup> establishes <math>d</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> radius <math>C_1 = 7.2</math></li> <li>•<sup>2</sup> <math>C_1C_2 = 13</math></li> <li>•<sup>3</sup> <math>d = (7.2 + 8) - 13 = 2.2</math></li> </ul>
2	ans: $90^\circ, 199.5^\circ, 340.5^\circ$ (5 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> subs for <math>\cos 2x^\circ</math></li> <li>•<sup>2</sup> multiplies and simplifies</li> <li>•<sup>3</sup> factorises</li> <li>•<sup>4</sup> finds two solutions</li> <li>•<sup>5</sup> finds third solution</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>3(1 - 2\sin^2 x^\circ) + 4\sin x^\circ - 1 = 0</math></li> <li>•<sup>2</sup> <math>-6\sin^2 x^\circ + 4\sin x^\circ + 2 = 0</math></li> <li>•<sup>3</sup> <math>2(3\sin x^\circ + 1)(\sin x^\circ - 1) = 0</math></li> <li>•<sup>4</sup> <math>\sin x^\circ = -\frac{1}{3}; x = 199.5^\circ, 340.5^\circ</math></li> <li>•<sup>5</sup> <math>\sin x^\circ = 1; x = 90^\circ</math></li> </ul>
	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	ans: $y = x^2 + \frac{6}{x} - 4$ (4 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to integrate</li> <li>•<sup>2</sup> integrates</li> <li>•<sup>3</sup> subs point</li> <li>•<sup>4</sup> solves for <math>C</math> and states function</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y = \int 2x - \frac{6}{x^2} dx</math></li> <li>•<sup>2</sup> <math>y = x^2 + \frac{6}{x} + C</math></li> <li>•<sup>3</sup> <math>3 = 2^2 + \frac{6}{2} + C</math></li> <li>•<sup>4</sup> <math>y = x^2 + \frac{6}{x} - 4</math></li> </ul>
(b)	ans: $p = 7$ (1 mark)	

●<sup>1</sup> subs point and solves for  $p$

●<sup>1</sup>  $p = 3^2 + \frac{6}{3} - 4 = 7$

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**4(a) ans: P(3, 0) (2 marks)**

●<sup>1</sup> knows to make function equal to 0  
●<sup>2</sup> solves for  $x$  and states coords of P

●<sup>1</sup>  $x^3 - x^2 - 5x - 3 = 0$   
●<sup>2</sup>  $x = 3; P(3, 0)$

**(b) ans:  $2y + 3x = 9$  (1 mark)**

●<sup>1</sup> subs info into formula for straight line

●<sup>1</sup>  $y = -\frac{3}{2}(x - 3)$

**(c) ans:  $y - 11x = 17$  (4 marks)**

●<sup>1</sup> knows to take derivative  
●<sup>2</sup> subs to find gradient  
●<sup>3</sup> subs to find point of contact  
●<sup>4</sup> subs into straight line formula

●<sup>1</sup>  $\frac{dy}{dx} = 3x^2 - 2x - 5$   
●<sup>2</sup>  $3(-2)^2 - 2(-2) - 5 = 11$   
●<sup>3</sup>  $y = (-2)^3 - (-2)^2 - 5(-2) - 3 = -5$   
●<sup>4</sup>  $y + 5 = 11(x + 2); y - 11x = 17$

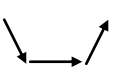
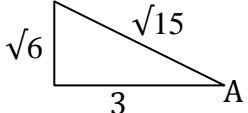
**(d) ans: Q(-1, 6) (3 marks)**

●<sup>1</sup> knows to use sim. eqs.  
●<sup>2</sup> solves for  $x$  and  $y$   
●<sup>3</sup> states coordinates of Q

●<sup>1</sup> evidence  
●<sup>2</sup>  $x = -1$  and  $y = 6$   
●<sup>3</sup> Q(-1, 6)

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	<b>ans: 143.3gu's (2 marks)</b> • <sup>1</sup> knows how to calculate answer • <sup>2</sup> answer	• <sup>1</sup> $0.924 \times 200$ • <sup>2</sup> 143.3gu's
(b)	<b>ans: 135.8 gu's (3 marks)</b>  • <sup>1</sup> sets up recurrence relation • <sup>2</sup> repeated calculations to answer • <sup>3</sup> repeated calculations to answer	• <sup>1</sup> $U_{n+1} = 0.92^4 U_n + 32$ • <sup>2</sup> 175.3[after 4 hours]; 157.6[after 8 hours] • <sup>3</sup> 144.9[after 12 hours]; 135.8[after 16 hours]
(c)	<b>ans: yes since lower limit is 80.8 (3 marks)</b>  • <sup>1</sup> knows to find limit • <sup>2</sup> finds limit • <sup>3</sup> realises lower limit is less than 100	• <sup>1</sup> $L = \frac{32}{1-0.92^4}$ • <sup>2</sup> $L = 112.8$ • <sup>3</sup> brightness would fall below 100 since lower limit is 80.8
6(a)	<b>ans: proof (3 marks)</b>  • <sup>1</sup> cross multiplies and multiplies out • <sup>2</sup> brings to LHS • <sup>3</sup> rearranges as required	• <sup>1</sup> $k(x^2 + 4) = x^2 - 2x + 1$ • <sup>2</sup> $kx^2 - x^2 + 2x + 4k - 1$ • <sup>3</sup> $(k - 1)x^2 + 2x + (4k - 1) = 0$
(b)	<b>ans: <math>k = \frac{5}{4}</math> (5 marks)</b>  • <sup>1</sup> states condition for equal roots • <sup>2</sup> states values of $a$ , $b$ and $c$ • <sup>3</sup> substitutes into $b^2 - 4ac$ • <sup>4</sup> multiplies out and simplifies • <sup>5</sup> solves for $k$	• <sup>1</sup> $b^2 - 4ac = 0$ for equal roots [stated/implied] • <sup>2</sup> $a = (k - 1); b = 2; c = (4k - 1)$ • <sup>3</sup> $2^2 - 4(k - 1)(4k - 1) = 0$ • <sup>4</sup> $20k - 16k^2 = 0$ • <sup>5</sup> $k = \frac{5}{4}$



	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	<b>ans: proof (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> finds expression for length of shed</li> <li>•<sup>2</sup> finds expression for area of g'house</li> <li>•<sup>3</sup> simplifies to correct form</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> length of shed = <math>\frac{3}{x}</math></li> <li>•<sup>2</sup> <math>A = (x + 3)(4 + \frac{3}{x}) - 3</math></li> <li>•<sup>3</sup> <math>A = 4x + 3 + 12 + \frac{9}{x} - 3 \rightarrow</math> answer</li> </ul>
(b)	<b>ans: 24 (5 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to equate derivative to 0</li> <li>•<sup>2</sup> prepares to differentiate</li> <li>•<sup>3</sup> differentiates</li> <li>•<sup>4</sup> solves for <math>x</math></li> <li>•<sup>5</sup> justifies answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 0</math></li> <li>•<sup>2</sup> <math>A(x) = 12 + 4x + 9x^{-1}</math></li> <li>•<sup>3</sup> <math>A'(x) = 4 - \frac{9}{x^2} = 0</math></li> <li>•<sup>4</sup> <math>4 - \frac{9}{x^2} = 0; x^2 = \frac{9}{4}; x = \frac{3}{2}</math></li> <li>•<sup>5</sup>  or other acceptable method</li> </ul>
8(a)	<b>ans: proof; <math>k = 10</math> (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> assembles facts in rt. triangle</li> <li>•<sup>2</sup> finds <math>\sin A</math></li> <li>•<sup>3</sup> rationalises denominator &amp; states <math>k</math></li> </ul>	 <ul style="list-style-type: none"> <li>•<sup>1</sup></li> <li>•<sup>2</sup> <math>\sin A = \frac{\sqrt{6}}{\sqrt{15}}</math></li> <li>•<sup>3</sup> <math>\frac{\sqrt{6}}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{10}}{5}; k = 10</math></li> </ul>
(b)	<b>ans: proof (3 marks)</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to change <math>\cos 2A</math></li> <li>•<sup>2</sup> substitutes</li> <li>•<sup>3</sup> simplifies to required answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\cos 2A = 1 - 2\sin^2 A</math></li> <li>•<sup>2</sup> <math>1 - 2(\frac{\sqrt{10}}{5})^2</math></li> <li>•<sup>3</sup> <math>1 - 2 \times \frac{10}{25} = 1 - \frac{4}{5} = \frac{1}{5}</math></li> </ul>

Total: 60 marks