

PRELIM 2009-10

INVERGORDON ACADEMY
MATHEMATICS DEPARTMENT

MATHEMATICS
HIGHER
Units 1 and 2
Paper 1
(Non-calculator)

TIME: 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB** pencil.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.



Read carefully

- 1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2009/2010 (Section A)**.
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- 3 Make sure you write your **name, class and teacher** on the answer sheet provided.
- 4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
- 5 There is **only one correct** answer to each question.
- 6 Rough working should **not** be done on your answer sheet.
- 7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

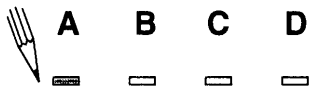
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k, 7)$ lies on this line, the value of k is

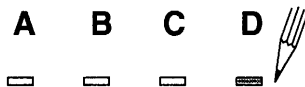
- A 2
- B 27
- C 1.5
- D -2

The correct answer is **A** \rightarrow 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

SECTION A

ALL questions should be attempted

1. A straight line with gradient 2 passes through the points $(-2,3)$ and $(-6, k)$.

The value of k is

- A 11
- B -5
- C 1
- D -13

2. Which of the following gives equal roots for the equation $x^2 + kx + 8 = 0$

- A $k = 8\sqrt{2}$
- B $k = 32$
- C $k = 6$
- D $k = 4\sqrt{2}$

3. The **rate of change** of the function $f(x) = \frac{3}{2x^2}$ when $x = 1$ is

- A -12
- B 3
- C $\frac{3}{2}$
- D -3

4. Given that $\cos A = \frac{\sqrt{2}}{5}$, the exact value of $\cos 2A$ is

- A $\frac{2\sqrt{2}}{5}$
- B $-\frac{17}{25}$
- C $-\frac{21}{25}$
- D cannot be found

5. $\int_0^{2a} 4x \, dx$ is

- A 4
- B $16a^2$
- C $8a^2$
- D $4a^2$

6. The gradient of the tangent to the circle $x^2 + y^2 + 6x - 4y - 4 = 0$ at the point (1,3) on the circumference is

- A $\frac{1}{4}$
- B -4
- C $-\frac{1}{4}$
- D $\frac{2}{5}$

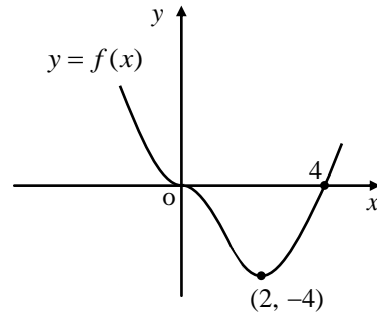
7. If $f(x) = 3x$ and $g(x) = x^2 - 6x$ then $g(f(1))$ is equal to

- A -15
- B -9
- C 0
- D 3

8. Three of the following are equal in value. Which is the exception?

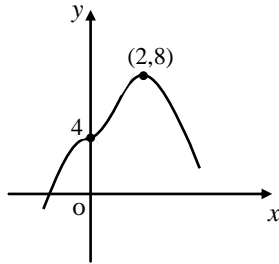
- A $\cos \frac{\pi}{3}$
- B $\sin \frac{5\pi}{6}$
- C $\cos \frac{4\pi}{3}$
- D $-\sin \frac{11\pi}{6}$

9. Part of the graph of the function $y = f(x)$ is shown opposite.

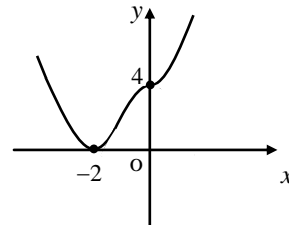


Which of the following graphs represents the related function $y = f(-x) + 4$?

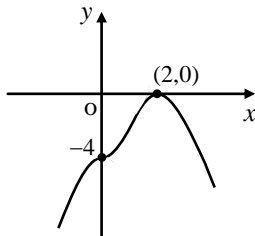
A



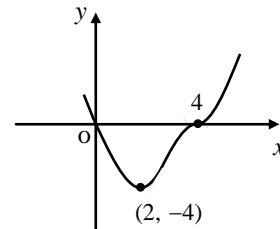
B



C



D



10. PQRS is a parallelogram. P, Q and R have coordinates $(-4, -3)$, $(-1,5)$ and $(7,7)$ respectively.

The coordinates of S are

A $(-11, -1)$

B $(4,0)$

C $(4, -1)$

D $(15,9)$

11. A recurrence relationship is defined as $U_{n+1} = 0.6U_n + b$.

If the limit of the recurrence relationship is 20, the value of b is

A 12

B 50

C $33\frac{1}{3}$

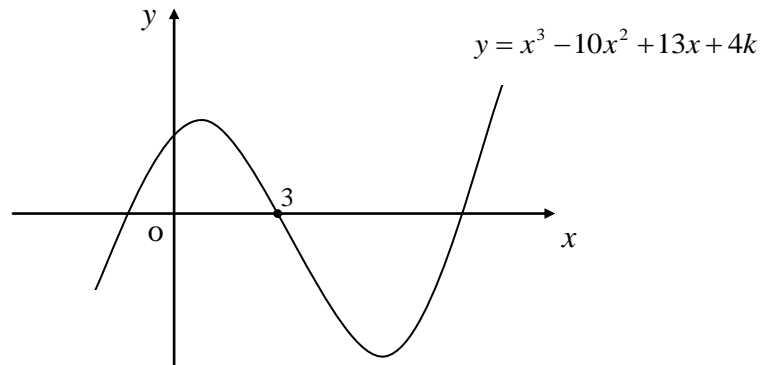
D 8

12. The minimum turning value of the function $f(x) = x^2 - 4x + 8$ is

- A 4
- B 2
- C 8
- D -4

13. The diagram below shows part of the curve with equation $y = x^3 - 10x^2 + 13x + 4k$.

The curve crosses the x -axis at $(3,0)$.



The value of k is

- A 12
- B 6
- C 0
- D -6

14. If $a^{\frac{1}{2}} = \sqrt{8} b^{\frac{3}{2}}$ then b equals

- A $\frac{a^{\frac{1}{3}}}{\sqrt{8}}$
- B $\frac{a^{\frac{1}{3}}}{2}$
- C $\frac{a^{\frac{3}{4}}}{\sqrt{8}}$
- D $\frac{a^{\frac{1}{3}}}{8}$

15. The equation of the circle, centre $(-5,6)$ and having the x -axis as a tangent is

A $(x+5)^2 + (y-6)^2 = 25$

B $(x-5)^2 + (y+6)^2 = 25$

C $(x+5)^2 + (y-6)^2 = 36$

D $(x-5)^2 + (y+6)^2 = 36$

16. $\sin 3x^\circ \sin 2x^\circ - \cos 3x^\circ \cos 2x^\circ$ equals

A $\sin x^\circ$

B $-\cos x^\circ$

C $\cos 5x^\circ$

D $-\cos 5x^\circ$

17. The range of values of x for which the function $f(x) = x^2 + 8x - 12$ is decreasing is

A $x < -4$

B $x > -4$

C $x < 4$

D $x > 4$

18. A function is defined as $f(x) = 3x^3 - 27x$.

It has a stationary point at

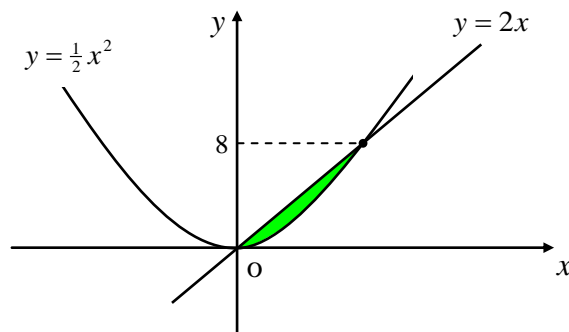
A $x = \sqrt{3}$ and $x = -\sqrt{3}$

B only $x = 3$

C only $x = \sqrt{3}$

D $x = 3$ and $x = -3$

19. The diagram, **which is not drawn to scale**, shows parts of the graphs of the curve $y = \frac{1}{2}x^2$ and the line $y = 2x$.



The shaded area above can be represented by

- A $\int_0^8 \left(2x - \frac{1}{2}x^2\right) dx$
- B $\int_0^4 \left(\frac{1}{2}x^2 - 2x\right) dx$
- C $\int_0^4 \left(2x - \frac{1}{2}x^2\right) dx$
- D $\int_0^8 \left(\frac{1}{2}x^2 + 2x\right) dx$
20. Given that $\sin x = a^2 - 1$ and $\cos x = a - 1$ then $\tan x$ equals

- A a
- B $a + 1$
- C $a^3 - a^2 - a + 1$
- D $\frac{1}{a + 1}$

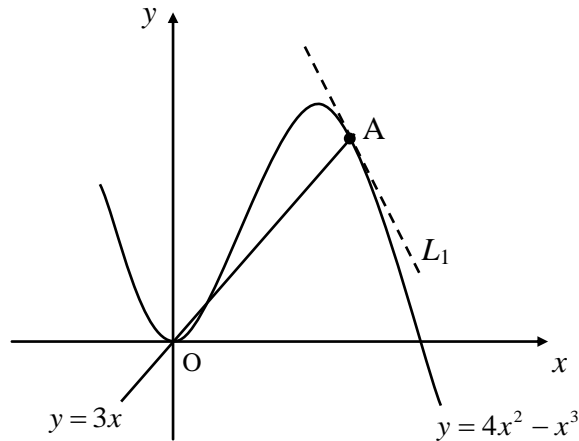
[END OF SECTION A]

SECTION B

ALL questions should be attempted

21. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve $y = 4x^2 - x^3$ and the straight line $y = 3x$.

The line intersects the curve at three points.



- (a) Find the coordinates of the point A. 4
- (b) The line L_1 is the tangent to the curve at A.
Establish the equation of the line L_1 . 4
22. Angle x is acute and is such that $\cos x = \frac{\sqrt{2}}{10}$.
- (a) Show clearly that the exact value of $\sin x$ is $\frac{7\sqrt{2}}{10}$. 3
- (b) Hence show that $\sin(x + \frac{\pi}{4}) = 0.8$. 4

23. An equation in x is given as

$$\frac{5x}{x^2 - k^2} = \frac{4}{x + k^2}, \text{ where } k \text{ is a constant and non-zero.}$$

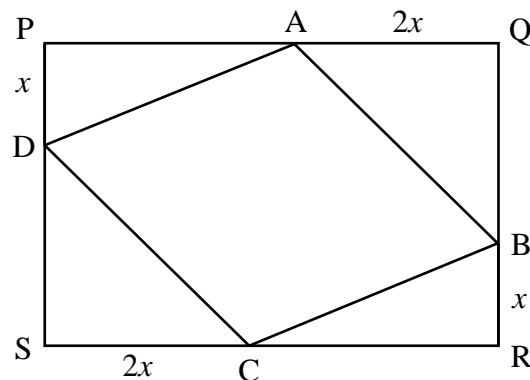
(a) Show that this equation can be written as

$$x^2 + 5k^2x + 4k^2 = 0 \quad 2$$

(b) Hence find the **two** values of k for which the equation $x^2 + 5k^2x + 4k^2 = 0$ has equal roots. 4

24. PQRS is a rectangle measuring **6 units by 4 units**.

Points A, B, C and D are points on the sides of the rectangle such that $AQ = SC = 2x$ and $PD = BR = x$ as shown.



(a) Show that the area of ABCD is given by the function $A(x) = 4x^2 - 14x + 24$. 4

(b) Hence find the value of x which minimises the area of ABCD and calculate this minimum area. 5

[END OF SECTION B]

[END OF QUESTION PAPER]

PRELIM 2009-10

INVERGORDON ACADEMY
MATHEMATICS DEPARTMENT

MATHEMATICS
HIGHER
Units 1 and 2
Paper 2

TIME: 1 hour 10 minutes

Read carefully

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

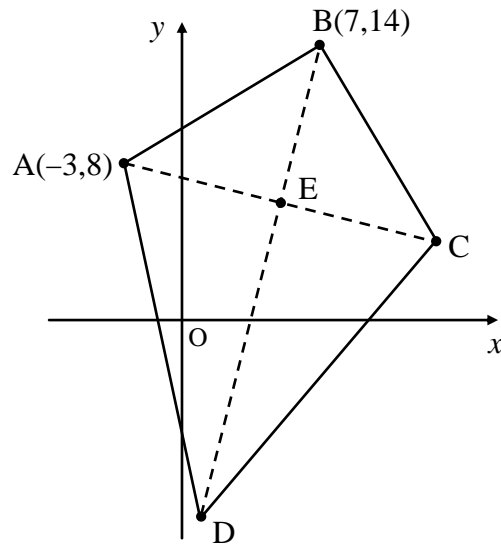
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

ALL questions should be attempted

1. Kite ABCD has two of its vertices at $A(-3,8)$ and $B(7,14)$ as shown.



- (a) Given that the equation of the longer diagonal BD is $y = 4x - 14$, find the equation of the short diagonal AC expressing your answer in the form $ax + by + c = 0$ and write down the values of a , b and c . **4**
- (b) Find the coordinates of E , the point of intersection of the two diagonals. **3**
- (c) Hence establish the coordinates of C . **2**

2. Two functions are defined on suitable domains as $f(x) = 4x + 1$ and $g(x) = \frac{1}{x-1}$.

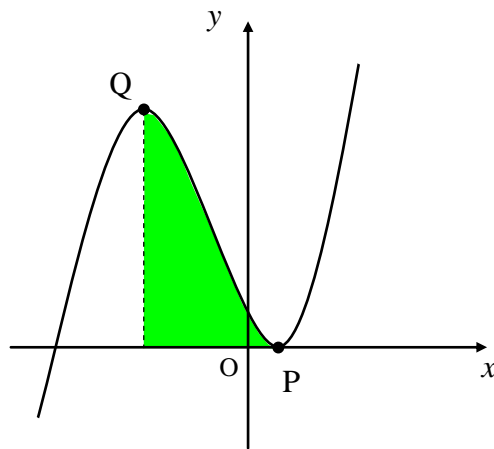
- (a) If $h(x) = f(g(x))$, show clearly that $h(x)$ can be written as

$$h(x) = \frac{x+3}{x-1}. \quad \mathbf{3}$$

- (b) Show that value of $h(\sqrt{5})$ can be expressed in the form $p + \sqrt{5}$ and write down the value of p . **4**

3. A function is defined on the set of real numbers as $f(x) = 2x^3 + 3x^2 - 12x + 7$.

Part of the graph of $y = f(x)$ is shown below.



- (a) Find the coordinates of the stationary points P and Q. 5
- (b) Calculate the shaded area in the diagram. 4

4. Solve algebraically the equation

$$15 \sin 2x^\circ = 10 \cos x^\circ \quad \text{for} \quad 0 \leq x < 360. \quad 5$$

5. A recurrence relationship is such that $U_{n+1} = \frac{a}{4}U_n + 12$.

- (a) If $U_0 = 16$ show clearly that $U_2 = a^2 + 3a + 12$. 2
- (b) Hence find a if $U_2 = 30$ and $a > 0$. 3
- (c) Explain why this sequence has a limit and find the limit. 3

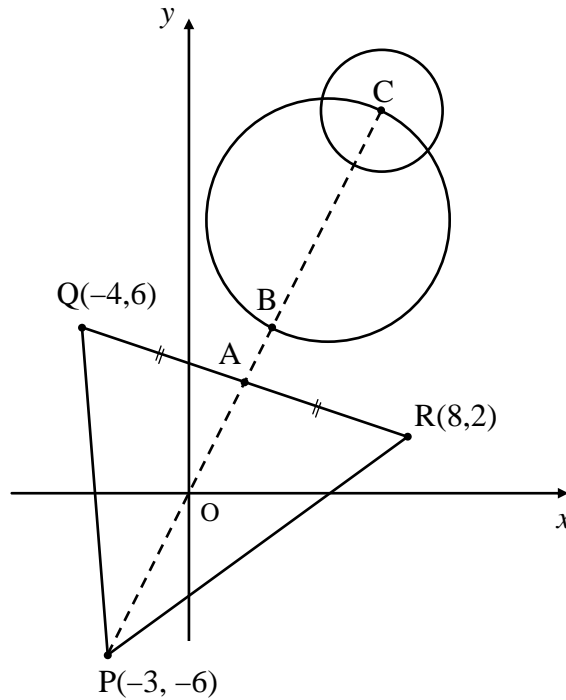
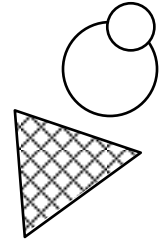
6. An ice-cream manufacturer has decided on a new logo for her company.

It consists of a triangle and two circles representing a wafer cone and two balls of ice cream.

Placed on a set of rectangular axes the logo is modelled in the diagram below.

The triangle has coordinates $P(-3, -6)$, $Q(-4, 6)$ and $R(8, 2)$.

A is the midpoint of QR .

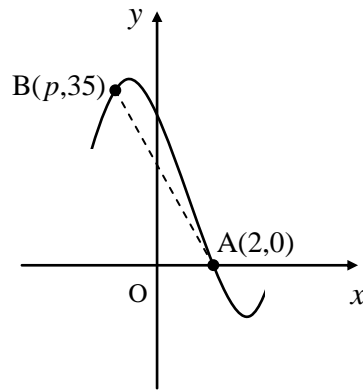


- (a) Find the equation of PA 3
- (b) When PA is extended it intersects with the larger circle at B and C .
 If the larger circle has as its equation $x^2 + y^2 - 10x - 20y + 105 = 0$,
 find the coordinates of C . 4
- (c) Given that C is the centre of the smaller circle and that its radius is
exactly half of the larger circle, find the equation of the smaller circle. 3

7. A curve has as its equation $y = x^3 - kx^2 - 16x + 32$.

Part of the graph of this curve is shown below.

The diagram is not drawn to scale.



- (a) If the curve crosses the x -axis at $A(2,0)$, find k . **3**
- (b) The point $B(p,35)$ also lies on this curve, find the value of p . **3**
- (c) Calculate the size of the angle between the line AB and the x -axis in the positive direction. **Give your answer to the nearest degree.** **2**

8. Given that $\int_0^a (2x) dx$ is equal to the **derivative** of $3a^2 - 9a$, find a . **4**

[**END OF QUESTION PAPER**]

Paper 1 - Section A - Answer Sheet

NAME :

CLASS :

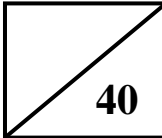
TEACHER :

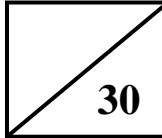
You should use an HB pencil.
 Erase all incorrect answers thoroughly.

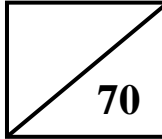
Indicate your choice of answer with a single mark as in this example →

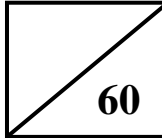
A	B	C	D
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

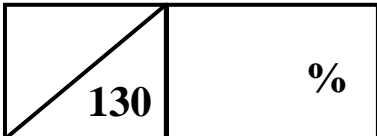
	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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18	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
19	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
20	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Section A 

Section B 

Total (P1) 

Total (P2) 

Overall Total 

Please make sure you have filled in all your details above before handing in this answer sheet.

- 1 B
- 2 D
- 3 D
- 4 C
- 5 C
- 6 B
- 7 B
- 8 C
- 9 B
- 10 C
- 11 D
- 12 A
- 13 B
- 14 B
- 15 C
- 16 D
- 17 A
- 18 A
- 19 C
- 20 B

	A	B	C	D
1	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
12	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
13	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
14	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
15	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
16	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
17	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
18	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
19	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
20	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	ans: A(3, 9) (4 marks) • ¹ equates 2 equations and collects to LHS • ² factorises • ³ solves for x chooses appropriate value • ⁴ substitutes and states point A	• ¹ $4x^2 - x^3 = 3x; 4x^2 - x^3 + 3x = 0$ • ² $x(x-3)(x-1) = 0$ • ³ $x = 0, 1, 3; x = 3$ • ⁴ $y = 3 \times 3 = 9; A(3, 9)$
	(b) ans: $y + 3x = 18$ (4 marks) • ¹ know to take derivative • ² knows to substitute • ³ evaluates to find gradient • ⁴ substitutes into $y - b = m(x - a)$	• ¹ $\frac{dy}{dx} = 8x - 3x^2$ • ² $8(3) - 3(3)^2$ • ³ $m = -3$ • ⁴ $y - 9 = -3(x - 3)$
22(a)	ans: proof (3 marks) • ¹ uses right triangle and finds third side • ² writes ratio of sin • ³ shows simplification to answer OR • ¹ knows to sub in appropriate formula • ² subs in formula and starts to simplify • ³ shows simplification to answer	• ¹ triangle with sides $\sqrt{2}$, 10 and $\sqrt{98}$ • ² $\sin x = \frac{\sqrt{98}}{10}$ • ³ $\sqrt{98} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$
	(b) ans: proof (4 marks) • ¹ knows to expand • ² correct expansion • ³ subs values • ⁴ simplifies to answer	• ¹ evidence of expansion • ² $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ • ³ $\frac{7\sqrt{2}}{10} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{10} \times \frac{1}{\sqrt{2}}$ • ⁴ $\frac{7\sqrt{2}}{10\sqrt{2}} + \frac{\sqrt{2}}{10\sqrt{2}} = \frac{8\sqrt{2}}{10\sqrt{2}} = \frac{8}{10} = 0.8$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	ans: proof (2 marks) <ul style="list-style-type: none"> •¹ cross multiplies •² multiplies brackets and collects terms 	<ul style="list-style-type: none"> •¹ $5x(x + k^2) = 4(x^2 - k^2)$ •² $5x^2 + 5k^2x = 4x^2 - 4k^2$ $5x^2 + 5k^2x - 4x^2 + 4k^2 = 0$
(b)	ans: $k = \pm \frac{4}{5}$ (4 marks) <ul style="list-style-type: none"> •¹ knows condition for equal roots •² substitutes values •³ factorises •⁴ solves and chooses values for k 	<ul style="list-style-type: none"> •¹ $b^2 - 4ac = 0$ [stated or implied] •² $(5k^2)^2 - 4 \times 1 \times 4k^2 = 0$ $25k^4 - 16k^2 = 0$ •³ $k^2(5k - 4)(5k + 4) = 0$ •⁴ $k = 0; \pm \frac{4}{5}$
24(a)	ans: proof (4 marks) <ul style="list-style-type: none"> •¹ finds expressions for missing dimensions •² finds area of triangles •³ subtracts from area of rectangle •⁴ simplifies to answer 	<ul style="list-style-type: none"> •¹ $(6 - 2x)$ and $(4 - x)$ •² $x(6 - 2x)$ and $2x(4 - x)$ •³ $24 - (6x - 2x^2 + 8x - 2x^2)$ •⁴ $24 - 6x + 2x^2 - 8x + 2x^2$
(b)	ans: $\frac{7}{4}; 11\frac{3}{4}$ (5 marks) <ul style="list-style-type: none"> •¹ knows to make derivative equal to 0 •² finds derivative a •³ solves for x and justifies •⁴ subs value to find area •⁵ answer 	<ul style="list-style-type: none"> •¹ $\frac{dy}{dx} = 0$ •² $\frac{dy}{dx} = 8x - 14 = 0$ •³ $x = \frac{7}{4}$; table of values or second derivative •⁴ $a = 4\left(\frac{7}{4}\right)^2 - 14\left(\frac{7}{4}\right) + 24$ •⁵ $11\frac{3}{4}$

Total: 70 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $a = 1; b = 4, c = -29$ (4 marks) <ul style="list-style-type: none"> •¹ finds gradient of BD •² finds gradient of AC •³ subs into $y - b = m(x - a)$ and rearranges •⁴ states values of a, b and c 	<ul style="list-style-type: none"> •¹ $m_{BD} = 4$ [from equation] •² $m_{AC} = -1/4$ •³ $y - 8 = -\frac{1}{4}(x + 3); x + 4y - 29 = 0$ •⁴ $a = 1; b = 4, c = -29$
(b)	ans: $E(5, 6)$ (3 marks) <ul style="list-style-type: none"> •¹ knows to use system of equations •² solves for x and y •³ states coordinates of E 	<ul style="list-style-type: none"> •¹ evidence of equating one variable •² $x = 5; y = 6$ •³ $E(5, 6)$
(c)	ans: $C(13, 4)$ (2 marks) <ul style="list-style-type: none"> •¹ appropriate method •² answer 	<ul style="list-style-type: none"> •¹ evidence of 'stepping out' or other method •² $C(13, 4)$
2(a)	ans: proof (3 marks) <ul style="list-style-type: none"> •¹ knows to substitute •² substitutes correctly •³ clearly simplifies to answer 	<ul style="list-style-type: none"> •¹ evidence of sub. one function in other •² $f\left(\frac{1}{x-1}\right) = \frac{4}{x-1} + 1$ •³ $\frac{4+x-1}{x-1} = \frac{x+3}{x-1}$
(b)	ans: $p = 2$ (4 marks) <ul style="list-style-type: none"> •¹ substitute for x •² knows to multiply by conjugate surd •³ multiplies and simplifies •⁴ states value of p 	<ul style="list-style-type: none"> •¹ $\frac{\sqrt{5} + 3}{\sqrt{5} - 1}$ •² $\frac{\sqrt{5} + 3}{\sqrt{5} - 1} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1}$ •³ $\frac{5 + 4\sqrt{5} + 3}{4} = \frac{8 + 4\sqrt{5}}{4} = 2 + \sqrt{5}$ •⁴ $p = 2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	ans: P(1, 0); Q(-2, 27) (5 marks) <ul style="list-style-type: none"> •¹ knows derivative = 0 at S.P. •² takes derivative and factorises •³ solves for x and chooses appropriate value •⁴ substitutes to find y – coordinate •⁵ states coordinates of P and Q 	<ul style="list-style-type: none"> •¹ $f'(x) = 0$ at SP [stated or implied] •² $6x^2 + 6x - 12 = 0$; $6(x+2)(x-1) = 0$ •³ $x = -2$ or 1 •⁴ $f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 7 = 27$ •⁵ P(1, 0); Q(-2, 27)
(b)	ans: 40.5 units² (4 marks) <ul style="list-style-type: none"> •¹ sets up integral •² integrates expression •³ substitutes values •⁴ evaluates 	<ul style="list-style-type: none"> •¹ $\int_{-2}^1 2x^3 + 3x^2 - 12x + 7 dx$ •² $\left[\frac{x^4}{2} + x^3 - 6x^2 + 7x \right]_{-2}^1$ •³ $\left(\frac{(1)^4}{2} + (1)^3 - 6(1)^2 + 7(1) \right) -$ •³ $\left(\frac{(-2)^4}{2} + (-2)^3 - 6(-2)^2 + 7(-2) \right)$ •⁴ 40.5 units²
4	ans: 19.5°, 90°, 160.5°, 270° (5 marks) <ul style="list-style-type: none"> •¹ subs for $\sin 2x^\circ$ •² multiplies and simplifies •³ factorises •⁴ finds two solutions •⁵ finds further two solutions 	<ul style="list-style-type: none"> •¹ $15(2 \sin x^\circ \cos x^\circ) \dots\dots$ •² $30 \sin x^\circ \cos x^\circ - 10 \cos x^\circ = 0$ •³ $10 \cos x^\circ (3 \sin x^\circ - 1) = 0$ •⁴ $\sin x^\circ = \frac{1}{3}$; $x = 19.5^\circ, 160.5^\circ$ •⁵ $\cos x^\circ = 0$; $x = 90^\circ, 270^\circ$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	ans: proof (2 marks) • ¹ substitutes U_0 and finds U_1 • ² substitutes U_1 and finds U_2	 • ¹ $U_1 = \frac{a}{4} \times 16 + 12 = 4a + 12$ • ² $U_1 = \frac{a}{4}(4a + 12) + 12 = a^2 + 3a + 12$
(b)	ans: $a = 3$ (3 marks) • ¹ equates U_2 to 30 • ² collects terms to LHS and factorises • ³ solves for x and discards	 • ¹ $a^2 + 3a + 12 = 30$ • ² $a^2 + 3a - 18 = 0; (a + 6)(a - 3) = 0$ • ³ $a = -6, 3; a = 3$
(c)	ans: 48 (3 marks) • ¹ knows condition for limit • ² knows how to find limit • ³ answer	 • ¹ limit exists since $-1 < \frac{3}{4} < 1$ • ² $L = \frac{12}{1 - 0.75} = \frac{12}{0.25}$ • ³ 48
6(a)	ans: $y = 2x$ (3 marks) • ¹ finds midpoint of QR • ² finds gradient of PA • ³ substitutes in $y - b = m(x - a)$	 • ¹ midpoint of QR = (2, 4) • ² $m_{PA} = \frac{4 + 6}{2 + 3} = 2$ • ³ $y - 4 = 2(x - 2); y = 2x$
(b)	ans: C(7, 14) (4 marks) • ¹ knows to substitute line into circle • ² multiplies and simplifies • ³ factorises and solves • ⁴ chooses appropriate value for x and subs	 • ¹ $x^2 + (2x)^2 - 10x - 20(2x) + 105 = 0$ • ² $5x^2 - 50x + 105 = 0$ • ³ $5(x - 3)(x - 7) = 0$ • ⁴ $x = 3, 7; x = 7, y = 14$
(c)	ans: $(x - 7)^2 + (y - 14)^2 = 5$ (3 marks) • ¹ finds radius of larger circle • ² finds radius of smaller circle • ³ subs into $(x - a)^2 + (y - b)^2 = r^2$	 • ¹ radius (large) = $\sqrt{25 + 100 - 105} = \sqrt{20}$ • ² radius (small) = $\sqrt{5}$ • ³ $(x - 7)^2 + (y - 14)^2 = 5$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	ans: $k = 2$ (3 marks) <ul style="list-style-type: none"> •¹ knows to use synthetic division •² makes remainder = 0 •³ solves for k 	<ul style="list-style-type: none"> •¹ evidence •² $8 - 4k = 0$ •³ $k = 2$
(b)	ans: $p = -3$ (3 marks) <ul style="list-style-type: none"> •¹ equates function to 35 •² collect terms to LHS and equates to 0 •³ uses synthetic division to find root 	<ul style="list-style-type: none"> •¹ $p^3 - 2p^2 - 16p + 32 = 35$ •² $p^3 - 2p^2 - 16p - 3 = 0$ •³ $p = -3$
(c)	ans: 98° (2 marks) <ul style="list-style-type: none"> •¹ finds gradient of AB •² takes \tan^{-1} and states angle 	<ul style="list-style-type: none"> •¹ $m_{AB} = \frac{35-0}{-3-2} = -7$ •² $\tan^{-1}(7) = 82^\circ$; angle = 98°
8	ans: $a = 3$ (4 marks) <ul style="list-style-type: none"> •¹ evaluates integral •² finds derivative •³ makes integral = derivative •⁴ factorises and solves 	<ul style="list-style-type: none"> •¹ $[x^2]_0^a = a^2$ •² $\frac{d}{da} = 6a - 9$ •³ $a^2 = 6a - 9$; $a^2 - 6a + 9 = 0$ •⁴ $(a - 3)(a - 3) = 0$; $a = 3$

Total: 60 marks