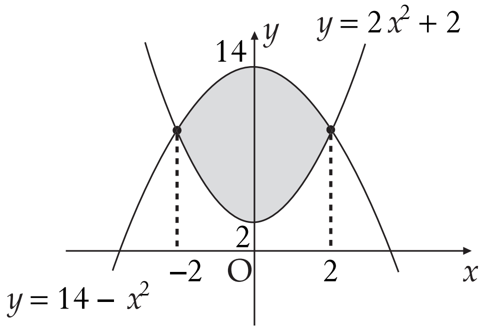
Area Between Curves

1. The diagram shows graphs with equations $y = 14 - x^2$ and $y = 2x^2 + 2$.



Which of the following represents the shaded area?

A. $\int_2^{14} (12 - 3x^2)\, dx$

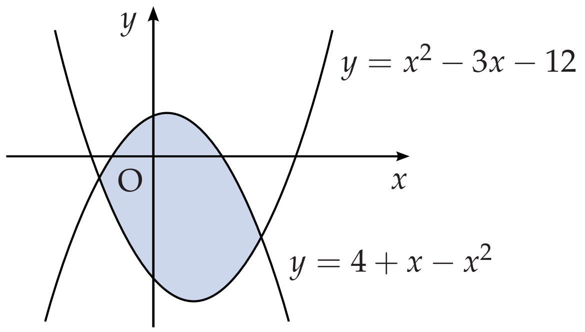
B. $\int_2^{14} (3x^2 - 12)\, dx$

C. $\int_{-2}^{2} (12 - 3x^2)\, dx$

D. $\int_{-2}^{2} (3x^2 - 12)\, dx$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.2	C	0	0	CN	C17	2010 P1 Q14

2. The parabolas $y = x^2 - 3x - 12$ and $y = 4 + x - x^2$ are shown in the diagram.



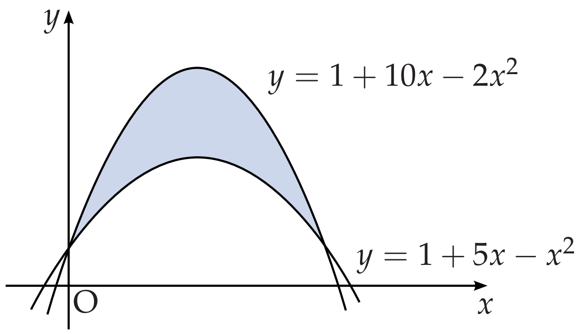
(*a*) Find the coordinates of the points of intersection of the parabolas. **3**

(*b*) Calculate the area enclosed between the parabolas. **6**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	3	C	CN	A25	$(-2,-2)$, $(4,-8)$	AT079
(*b*)	6	C	CN	C17	72 sq. units	

•1 ss: know to equate	•1 $x^2 - 3x - 12 = 4 + x - x^2$
•2 pd: solve quadratic equation	•2 $x = -2, 4$
•3 ic: complete coordinates	•3 $(-2,-2)$, $(4,-8)$
•4 ss: know to integrate "upper − lower"	•4 $\int (4 + x - x^2 - (x^2 - 3x - 12))\, dx$
•5 ic: use correct limits	•5 $\int_{-2}^{4} \cdots dx$
•6 pd: simplify integrand	•6 $\int_{-2}^{4} (-2x^2 + 4x + 16) dx$
•7 pd: integrate	•7 $[-\frac{2}{3}x^3 + 2x^2 + 16x]_{-2}^{4}$
•8 ic: substitute limits	•8 $(-\frac{2}{3}(4)^3+2(4)^2+16(4))-(-\frac{2}{3}(-2)^3+2(-2)^2+16(-2))$
•9 pd: complete	•9 72

[SQA] 3. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**

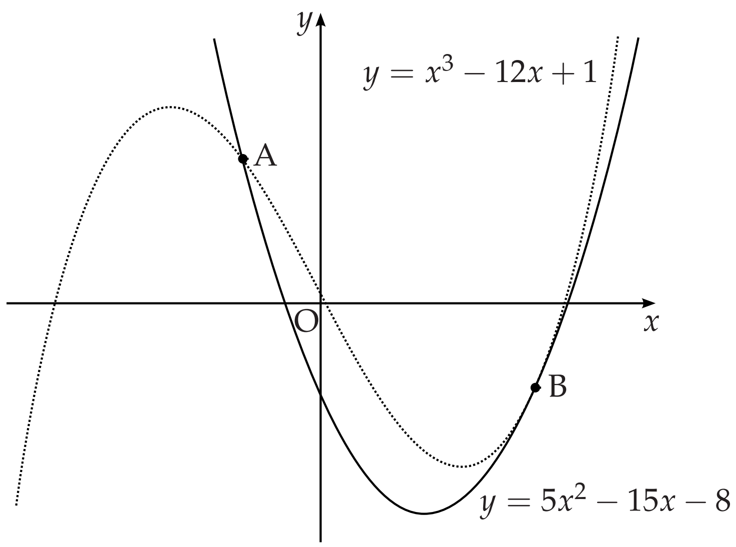


Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	6	C	CN	C17	$20\frac{5}{6}$	2002 P2 Q5

•1 ss: find intersections	•1 $1 + 10x - 2x^2 = 1 + 5x - x^2$
•2 ss: know to find limits	•2 $x = 0, 5$ **and** $\int_0^5 ()$
•3 ss: know to integrate (upper − lower)	•3 $\int ((1 + 10x - 2x^2) - (1 + 5x - x^2))\, dx$
•4 pd: simplify	•4 $\int (5x - x^2)\, dx$
•5 pd: integrate	•5 $\frac{5}{2}x^2 - \frac{1}{3}x^3$
•6 pd: process limits	•6 $20\frac{5}{6}$

[SQA] 4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

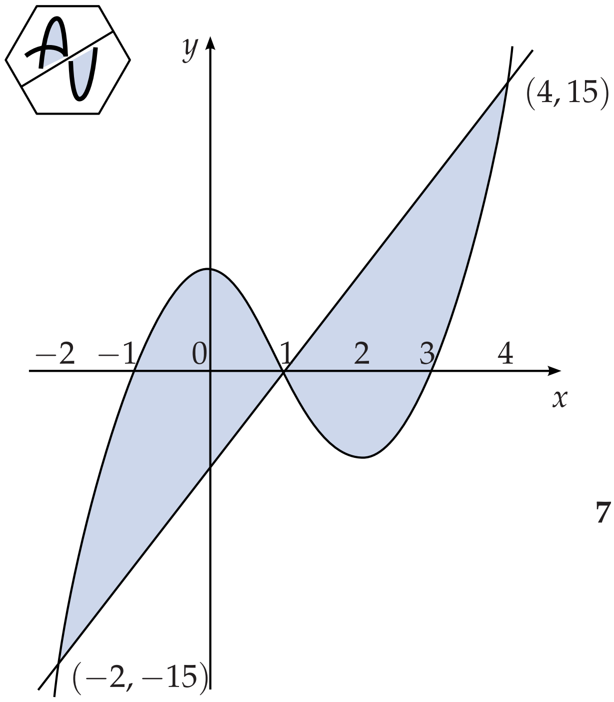
Find the area enclosed between the two curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*ai*)	4	C	NC	C4	$x = \frac{1}{3}$ and $x = 3$	2000 P1 Q4
(*aii*)	1	C	NC	CGD	parallel and coincident	
(*b*)	5	C	NC	C17	$21\frac{1}{3}$	

•1 ss: know to diff. and equate	•1 find derivatives and equate
•2 pd: differentiate	•2 $3x^2 - 12$ **and** $10x - 15$
•3 pd: form equation	•3 $3x^2 - 10x + 3 = 0$
•4 ic: interpret solution	•4 $x = 3,\ x = \frac{1}{3}$
•5 ic: interpret diagram	•5 tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident
•6 ss: know how to find area between curves	•6 $\int(\text{cubic} - \text{parabola})$ **or** $\int(\text{cubic}) - \int(\text{parabola})$
•7 ic: interpret limits	•7 $\int_{-1}^{3} \cdots dx$
•8 pd: form integral	•8 $\int(x^3 - 5x^2 + 3x + 9)dx$ or equiv.
•9 pd: process integration	•9 $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^{3}$ or equiv.
•10 pd: process limits	•10 $21\frac{1}{3}$

[SQA] 5. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.



The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

Calculate the total shaded area. **7**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	7	C	CN	C17	$40\frac{1}{2}$ units^2	2001 P2 Q8

•[1] ss: express in standard form	•[1] $y = x^3 - 3x^2 - x + 3$
•[2] ss: split area and integrate	•[2] $\int_1^4 (\ldots)dx$ **or** $\int_{-2}^1 (\ldots)dx$
•[3] ss: subtract functions	•[3] $\int \left[(5x-5) - (x^3 - 3x^2 - x + 3)\right] dx$ **or** $\int \left[(x^3 - 3x^2 - x + 3) - (5x-5)\right] dx$
•[4] pd: process	•[4] $\int (-x^3 + 3x^2 + 6x - 8)dx$
•[5] pd: process	•[5] $\left[-\frac{1}{4}x^4 + x^3 + 3x^2 - 8x\right]$
•[6] pd: process	•[6] $20\frac{1}{4}$ **or** $-20\frac{1}{4}$ *depending on chosen integrals*
•[7] ic: use symmetry or otherwise for total area	•[7] $40\frac{1}{2}$

[SQA] 6.

(*a*) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. **2**

(*b*) Find the area completely enclosed between these two curves. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	2	C	NC	A25		1990 P1 Q13
(*b*)	3	C	NC	C17		

•1 $2x^2 = 4x - 2x^2$ *or* $y = 4 - y$

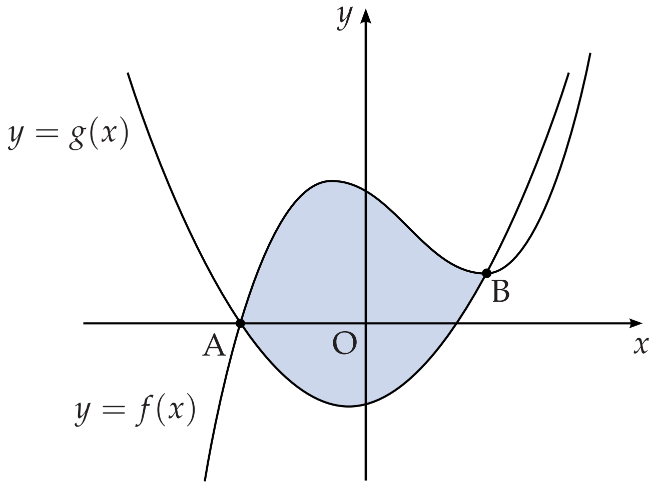
•2 $x = 1$ *and* $x = -1$

•3 $\int_{-1}^{1} \left(4 - 2x^2 - 2x^2\right) dx$

•4 $4x - \frac{2}{3}x^3 - \frac{2}{3}x^3$

•5 $5\frac{1}{3}$

7. The graphs of the functions $f(x) = x^3 - 2x^2 - 3x + 10$ and $g(x) = 2x^2 + x - 6$ are shown in the diagram.



The graphs intersect at the points A and B, and also where $x = 4$.

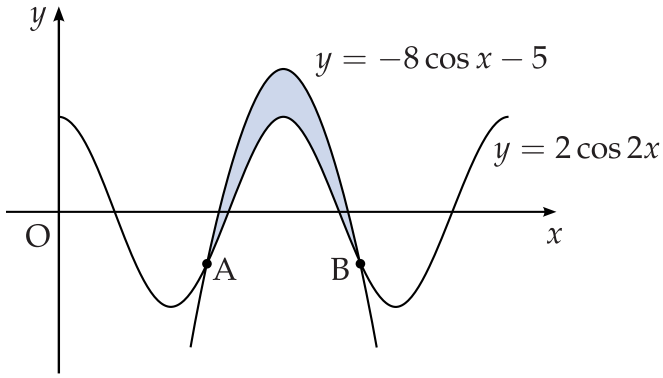
(*a*) Find the coordinates of A and B. **5**

(*b*) Calculate the shaded area enclosed between the curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	5	B	CN	A25	$A(-2, 0)$, $B(2, 4)$	AT014
(*b*)	5	C	CN	C17	$42\frac{2}{3}$	

•1 ss: form equation	•1 $x^3 - 2x^2 - 3x + 10 = 2x^2 + x - 6$
•2 ss: start to factorise	•2 $(x - 2) \cdots$
•3 pd: process	•3 $\cdots (x - 4)(x + 2) = 0$
•4 pd: identify x-coordinates	•4 $x = -2, 2, 4$
•5 ic: state coordinates	•5 $A(-2, 0)$, $B(2, 4)$
•6 ss: know how to find area	•6 $\int \left((x^3 - 2x^2 - 3x + 10) - (2x^2 + x - 6) \right) dx$
•7 ic: interpret limits	•7 $\int_{-2}^{2} \cdots dx$
•8 pd: form integral	•8 $\int_{-2}^{2} (x^3 - 4x^2 - 4x + 16)\, dx$
•9 pd: process integration	•9 $\left[\frac{1}{4}x^4 - \frac{4}{3}x^3 - 2x^2 + 16x \right]_{-2}^{2}$
•10 pd: process limits	•10 $\frac{128}{3}$ or $42\frac{2}{3}$

8. The diagram shows the graphs of $y = 2\cos 2x$ and $y = -8\cos x - 5$ on the interval $0 \leq x \leq 2\pi$.



The graphs intersect at the points A and B.

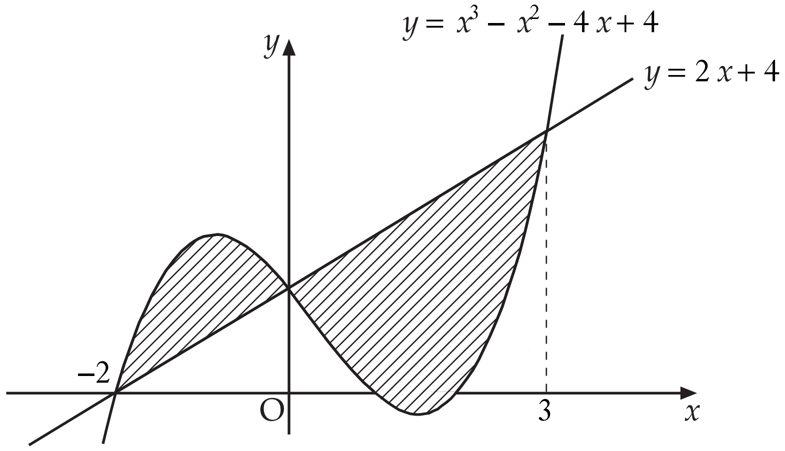
(*a*) Find the x-coordinates of A and B. **6**

(*b*) Show that the shaded area is $7\sqrt{3} - \frac{10\pi}{3}$ square units. **6**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(*a*)	6	C	CN	T7, T10, T3	$x_A = \frac{2\pi}{3}$, $x_B = \frac{4\pi}{3}$	AT056
(*b*)	6	C	CN	C17, C23, T3	proof	

•1 ss: equate	•1 $2\cos 2x = -8\cos x - 5$
•2 ss: use double angle formula	•2 $2(2\cos^2 x - 1) + 8\cos x + 5 = 0$
•3 pd: obtain standard form	•3 $4\cos^2 x + 8\cos x + 3 = 0$
•4 pd: factorise	•4 $(2\cos x + 3)(2\cos x + 1) = 0$
•5 pd: process	•5 no solutions **and** $x = \frac{2\pi}{3}, \frac{4\pi}{3}$
•6 ic: interpret solutions	•6 $x_A = \frac{2\pi}{3}$, $x_B = \frac{4\pi}{3}$
•7 ss: know to integrate	•7 $\int(-8\cos x - 5 - 2\cos 2x\,dx)$
•8 ss: use correct limits	•8 $\int_{2\pi/3}^{4\pi/3} \cdots dx$
•9 pd: integrate	•9 $[-\sin 2x - 8\sin x - 5x]_{2\pi/3}^{4\pi/3}$
•10 pd: process limits	•10 $7\sin\frac{2\pi}{3} - 7\sin\frac{4\pi}{3} - \frac{10\pi}{3}$
•11 pd: process exact values	•11 $7\frac{\sqrt{3}}{2} - 7(-\frac{\sqrt{3}}{2}) - \frac{10\pi}{3}$
•12 ic: complete	•12 $7\sqrt{3} - \frac{10\pi}{3}$

9. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.



Calculate the total shaded area. **10**

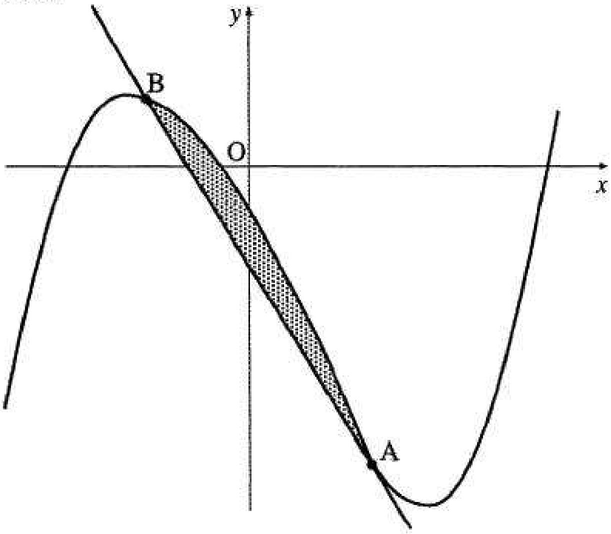
Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	10	B	CN	C17	$21\frac{1}{12}$	2011 P2 Q4

•[1] ss: know to integrate	•[1] $\int \cdots$ or attempt integration
•[2] ic: know to deal with areas on each side of y-axis	•[2] evidence of treating areas separately
•[3] ic: interpret limits of one area	•[3] e.g. $\int_0^3$
•[4] ic: use "upper − lower"	•[4] $(2x+4) - (x^3 - x^2 - 4x + 4)$
•[5] pd: integrate	•[5] $3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
•[6] ic: substitute in limits	•[6] $\left(3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4\right)$
•[7] pd: evaluate the area on one side	•[7] $\frac{63}{4}$
•[8] ss: interpret integrand with limits of the other area	•[8] $\int_{-2}^{0} (x^3 - x^2 - 4x + 4) - (2x+4)\,dx$
•[9] pd: evaluate the area on the other side	•[9] $\frac{16}{3}$
•[10] ic: state total area	•[10] $21\frac{1}{12}$ or $\frac{253}{12}$ or $21{\cdot}1$

[SQA] 10. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, − 8).

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	5	C	CN	C4, G3, A23		1996 P2 Q8
(*a*)	3	A/B	CN	C4, G3, A23		
(*b*)	3	A/B	CN	C17		

(a)

•1 *strat:* $\frac{dy}{dx} = \ldots$

•2 $\frac{dy}{dx} = 3x^2 - 2x - 6$

•3 $m_{tgt} = -5$

•4 $y + 8 = -5(x - 1)$

•5 *strat:* attempt to simplify and equate y's

•6 $x^3 - x^2 - x + 1 = 0$

•7 *strat:* *e.g.* try to factorise

•8 $B = (-1, 2)$

(b)

•9 $\int \left(x^3 - x^2 - 6x - 2\right) - (-5x - 3)\ dx$

•10 $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$

•11 $1\frac{1}{3}$

[END OF QUESTIONS]