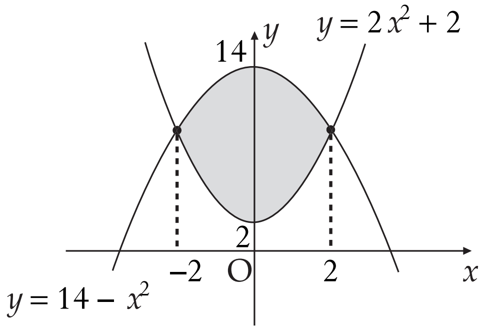
Area Between Curves

1. The diagram shows graphs with equations $y = 14 - x^2$ and $y = 2x^2 + 2$.



Which of the following represents the shaded area?

A. $\int_2^{14} (12 - 3x^2)\, dx$

B. $\int_2^{14} (3x^2 - 12)\, dx$

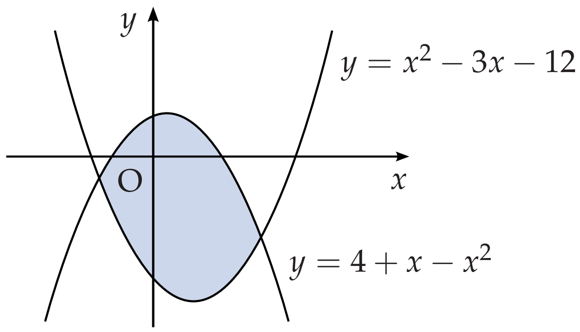
C. $\int_{-2}^{2} (12 - 3x^2)\, dx$

D. $\int_{-2}^{2} (3x^2 - 12)\, dx$ **2**

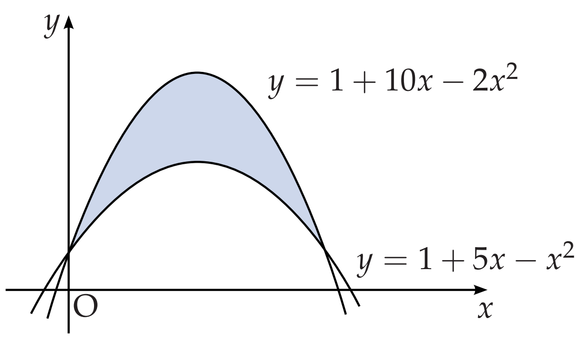
2. The parabolas $y = x^2 - 3x - 12$ and $y = 4 + x - x^2$ are shown in the diagram.

(*a*) Find the coordinates of the points of intersection of the parabolas. **3**

(*b*) Calculate the area enclosed between the parabolas. **6**



[SQA] 3. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**



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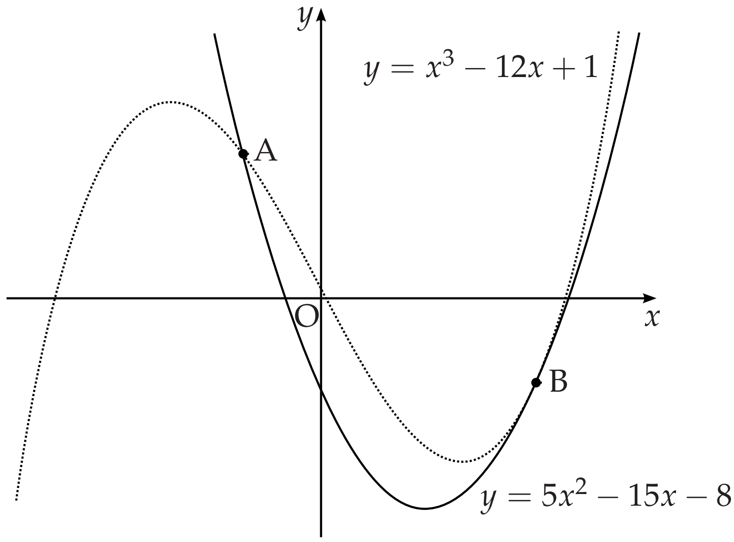
[SQA] 4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

Find the area enclosed between the two curves. **5**

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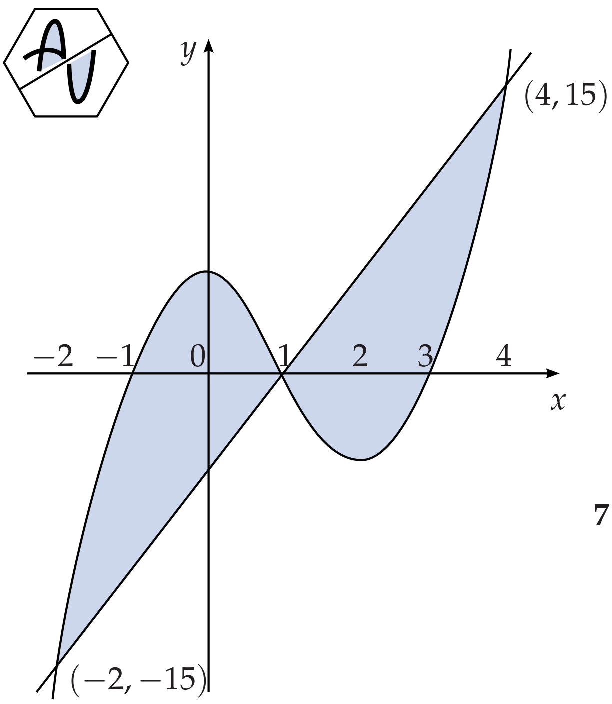
[SQA] 5. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

Calculate the total shaded area. **7**



[SQA] 6.

(*a*) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. **2**

(*b*) Find the area completely enclosed between these two curves. **3**

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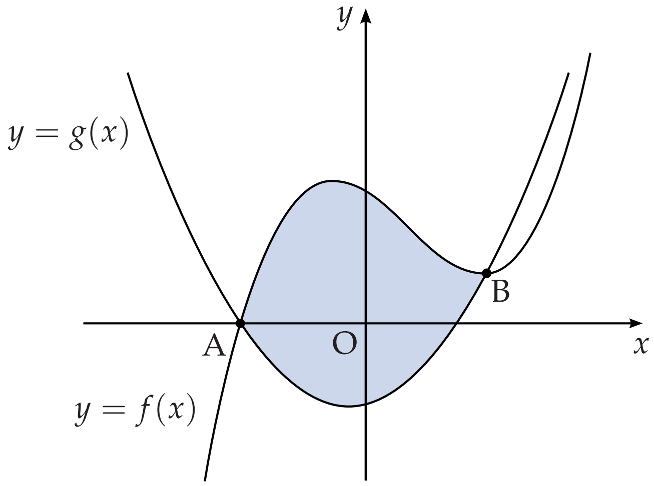
7. The graphs of the functions $f(x) = x^3 - 2x^2 - 3x + 10$ and $g(x) = 2x^2 + x - 6$ are shown in the diagram.

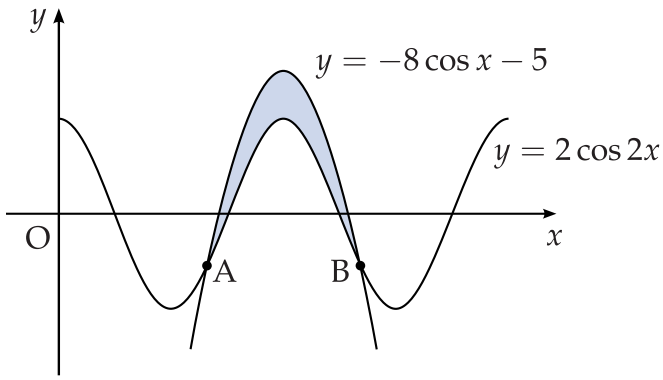


The graphs intersect at the points A and B, and also where $x = 4$.

(a) Find the coordinates of A and B. **5**

(b) Calculate the shaded area enclosed between the curves. **5**

8. The diagram shows the graphs of $y = 2\cos 2x$ and $y = -8\cos x - 5$ on the interval $0 \leq x \leq 2\pi$.



The graphs intersect at the points A and B.

(a) Find the x-coordinates of A and B. **6**

(b) Show that the shaded area is $7\sqrt{3} - \frac{10\pi}{3}$ square units. **6**

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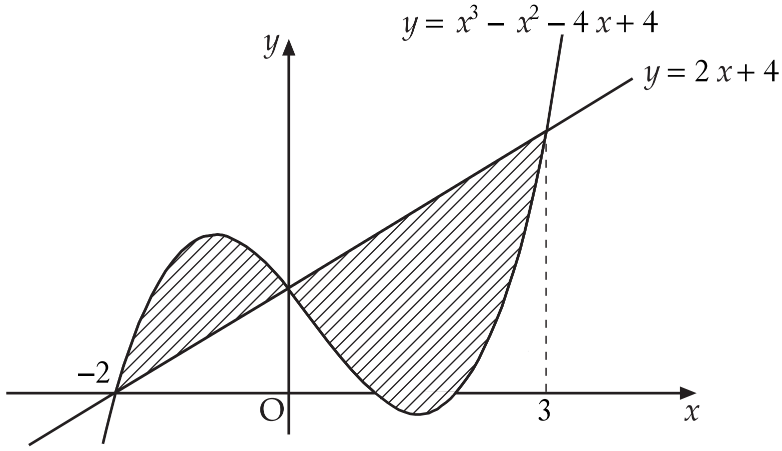
9. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.

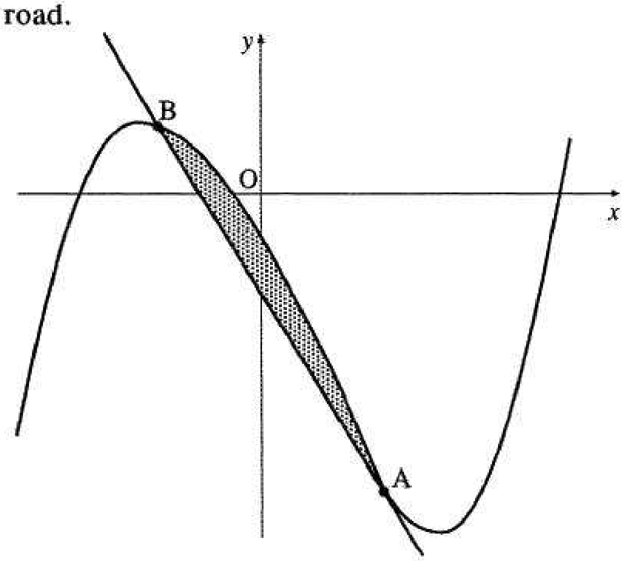


Calculate the total shaded area. **10**

[SQA] 10. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A$(1, -8)$.

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



[END OF QUESTIONS]

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