FURTHER CALCULUS SOLUTIONS

1. a) $y = 2x^3 - 3\cos x$

$\frac{dy}{dx} = 6x^2 + 3\sin x$

b) $y = (4-3x)^2$

$\frac{dy}{dx} = 2(4-3x).(-3)$

$= -6(4-3x)$

2. $\int \sqrt{(1-3x)}\,dx$

$= \int (1-3x)^{1/2}\,dx$

$= \frac{(1-3x)^{3/2}}{\frac{3}{2}.(-3)} + C$

$= -\frac{2}{9}\sqrt{(1-3x)^3} + C$

3. a) $\int_{-1}^{0} (3x+2)^3\,dx$

$= \left[\frac{(3x+2)^4}{4.3}\right]_{-1}^{0}$

$= \left[\frac{(3x+2)^4}{12}\right]_{-1}^{0}$

$= \left(\frac{(3(0)+2)^4}{12}\right) - \left(\frac{(3(-1)+2)^4}{12}\right)$

$= \frac{16}{12} - \frac{1}{12}$

$= \frac{15}{12}$

$= \frac{5}{4}$

b) $\int_{0}^{\frac{\pi}{2}} \sin 2x\,dx$

$= \left[-\frac{\cos 2x}{2}\right]_{0}^{\frac{\pi}{2}}$

$= \left(-\frac{1}{2}\cos 2\left(\frac{\pi}{2}\right)\right) - \left(-\frac{1}{2}\cos 2(0)\right)$

$= \left(-\frac{1}{2}.(-1)\right) - \left(-\frac{1}{2}.1\right)$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

4. $f(x) = \frac{1}{(1-2x)^2} + \sin 3x$

$= (1-2x)^{-2} + \sin 3x$

$f'(x) = -2(1-2x)^{-3}.(-2) + 3\cos 3x$

$= \frac{4}{(1-2x)^3} + 3\cos 3x$

5. $\frac{d}{dx}(\cos^2 x - \sin^2 x) = \frac{d}{dx}((\cos x)^2 - (\sin x)^2)$

$= 2(\cos x).(-\sin x) - 2(\sin x).\cos x$

$= -2\sin x\cos x - 2\sin x\cos x$

$= -4\sin x\cos x$

$\left(\begin{array}{l} = -2.2\sin x\cos x \\ = -2\sin 2x \end{array}\right)$

$\frac{d}{dx}(\cos^2 x - \sin^2 x) = \frac{d}{dx}(\cos 2x)$

$= -2\sin 2x$

6. $f(x) = \cos 2x - 3\sin 4x$

$f'(x) = -2\sin 2x - 3\cos 4x.4$

$= -2\sin 2x - 12\cos 4x$

$= -2(\sin 2x + 6\cos 4x)$

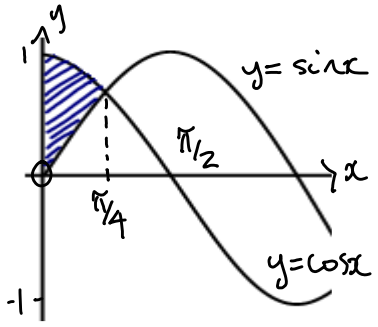
$f'\left(\frac{\pi}{6}\right) = -2\left(\sin 2\left(\frac{\pi}{6}\right) + 6\cos 4\left(\frac{\pi}{6}\right)\right)$

$= -2\left(\sin\frac{\pi}{3} + 6\cos\frac{2\pi}{3}\right)$

$= -2\left(\frac{\sqrt{3}}{2} + 6.\left(-\frac{1}{2}\right)\right)$

$= -2\left(\frac{\sqrt{3}}{2} - 3\right)$

$= 6 - \sqrt{3}$

7. 

$\int_0^{\pi/4} (\cos x - \sin x)\,dx$

$= \left[\sin x + \cos x\right]_0^{\pi/4}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1)$

$= \frac{2}{\sqrt{2}} - 1$

$= \sqrt{2} - 1$