INTEGRATION SOLUTIONS

1a) $\int (1-x)\,dx$

$= x - \frac{x^2}{2} + c$

b) $\int (3x^2 + 4x + 5)\,dx$

$= x^3 + 2x^2 + 5x + c$

c) $\int (1-3x)^2\,dx$

$= \int (1 - 6x + 9x^2)\,dx$

$= x - 3x^2 + 3x^3 + c$

d) $\int \left(x - \frac{1}{x}\right)^2 dx$

$= \int (x - x^{-1})(x - x^{-1})\,dx$

$= \int (x^2 - 2x^0 + x^{-2})\,dx$

$= \int (x^2 - 2 + x^{-2})\,dx$

$= \frac{x^3}{3} - 2x + \frac{x^{-1}}{-1} + c$

$= \frac{x^3}{3} - 2x - \frac{1}{x} + c$

e) $\int \frac{x^2 + 2}{\sqrt{x}}\,dx$

$= \int \left(\frac{x^2}{x^{1/2}} + \frac{2}{x^{1/2}}\right) dx$

$= \int \left(x^{3/2} + 2x^{-1/2}\right) dx$

$= \frac{x^{5/2}}{5/2} + \frac{2x^{1/2}}{1/2} + c$

$= \frac{2}{5}\sqrt{x^5} + 4\sqrt{x} + c$

2. $\frac{dy}{dx} = 3x^2 - 10x \qquad (-1, 0)$

$y = x^3 - 5x^2 + c$

when $x = -1,\ y = 0$

$\Rightarrow \quad 0 = (-1)^3 - 5(-1)^2 + c$

$0 = -1 - 5 + c$

$6 = c$

Particular solution is $y = x^3 - 5x^2 + 6$

3. $\int_1^3 \left(x^2 - \frac{1}{x^2}\right) dx$

$= \int_1^3 (x^2 - x^{-2})\, dx$

$= \left[\frac{x^3}{3} - \frac{x^{-1}}{(-1)}\right]_1^3$

$= \left[\frac{x^3}{3} + \frac{1}{x}\right]_1^3$

$= \left(\frac{(3)^3}{3} + \frac{1}{(3)}\right) - \left(\frac{(1)^3}{3} + \frac{1}{(1)}\right)$

$= \left(\frac{27}{3} + \frac{1}{3}\right) - \left(\frac{1}{3} + 1\right)$

$= \frac{28}{3} - \frac{4}{3}$

$= \frac{24}{3}$

$= \underline{\underline{8}}$

4. $\int_8^c x^{-\frac{2}{3}}\, dx = 3$

$\left[\frac{x^{1/3}}{1/3}\right]_8^c = 3$

$\left[3\sqrt[3]{x}\right]_8^c = 3$

$3\sqrt[3]{c} - 3\sqrt[3]{(8)} = 3$

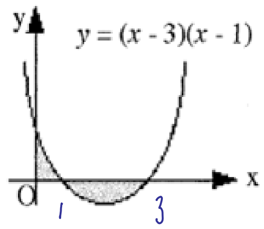
$\sqrt[3]{c} - \sqrt[3]{8} = 1$

$\sqrt[3]{c} - 2 = 1$

$\sqrt[3]{c} = 3$

$\underline{\underline{c = 27}}$

5.



$y = (x-3)(x-1)$

$y = x^2 - 4x + 3$

on x-axis, $y = 0$

$\Rightarrow (x-3)(x-1) = 0$

$\Rightarrow x = 1, 3$

$\int_0^1 (x^2 - 4x + 3)\, dx$

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1$

$= \left(\frac{(1)^3}{3} - 2(1)^2 + 3(1)\right) - 0$

$= \left(\frac{1}{3} - 2 + 3\right)$

$= \frac{4}{3}$

$\int_1^3 (x^2 - 4x + 3)\, dx$

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3$

$= \left(\frac{(3)^3}{3} - 2(3)^2 + 3(3)\right) - \left(\frac{(1)^3}{3} - 2(1)^2 + 3(1)\right)$

$= (9 - 18 + 9) - \left(\frac{1}{3} - 2 + 3\right)$

$= -\frac{4}{3}$

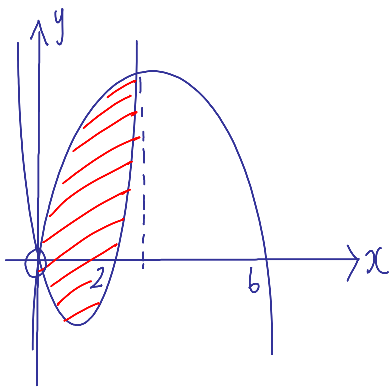
Total area $= 2 \times \frac{4}{3} = \frac{8}{3}$ units^2

6a) $y = x^2 - 2x$

$y = x(x-2)$

$y = 6x - x^2$

$y = x(6-x)$



b) for points of intersection, $y = y$

$$\Rightarrow \quad x^2 - 2x = 6x - x^2$$
$$2x^2 - 8x = 0$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$\Rightarrow \quad x = 0, 4$$

$$\int_0^4 \left(6x - x^2 - (x^2 - 2x)\right) dx$$
$$= \int_0^4 (8x - 2x^2)\, dx$$
$$= \left[4x^2 - \frac{2x^3}{3}\right]_0^4$$
$$= \left(4(4)^2 - \frac{2(4)^3}{3}\right) - 0$$
$$= 64 - \frac{128}{3}$$
$$= \frac{192}{3} - \frac{128}{3}$$
$$= \underline{\underline{\frac{64}{3} \text{ units}^2}}$$